UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

THINKING SKILLS **9694/33**

Paper 3 Problem Analysis and Solution **October/November 2012**

1 hour 30 minutes

Additional Materials: Answer Booklet/Paper
Electronic Calculator

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.
Calculators should be used where appropriate.

Answer **all** the questions.
Start each question on a new answer sheet.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **7** printed pages and **1** blank page.

1 *Study the information below and answer the questions. Show your working.*

New members of a company are invited to play a simple strategy game as part of their induction day. Each game consists of 12 rounds, played between two players. Both players choose a sequence of 12 letters made up of As, Bs and Cs. They then compare their lists letter by letter. Points are awarded as follows.

Letters played	*2 points for…*	*0 points for…*
A and B	A	B
B and C	B	C
C and A	C	A

When players play the same letter, they both get one point.

Three of the players have decided not to change their sequences from game to game.

Luke plays B A B A C A C A A A C A.
Maggie repeats the sequence A B C over and over again.
Nigel plays A B A B B B C A C A B C.

(a) What will Maggie's and Nigel's scores be when they each play a game against Luke? [2]

(b) In some games, one player may have scored enough points to ensure a win before all rounds have been completed. In which of the three games between Luke, Maggie and Nigel does this occur, and after which round? [2]

A fourth player, Ophelia, takes a different approach, which involves always starting with A, and subsequently playing whatever letter her opponent played in the previous round.

(c) When Luke, Maggie and Nigel each play against Ophelia, which of the three scores the least and what is that score? [2]

(d) Pedro is going to play against Ophelia. Give a sequence of letters that he could play which would ensure that she scored nothing. [1]

(e) Quentin joins in and plays against Luke, Maggie and Nigel. Suggest a sequence of 12 letters which would produce the maximum total score for Quentin over the three games. [3]

2 *Study the information below and answer the questions. Show your working.*

In order to check that they have recorded credit card numbers correctly, businesses can do the following:

- Change the digits in the **odd positions** (counting from the left: first, third, etc.) according to the table below.

Original	0	1	2	3	4	5	6	7	8	9
Changed	0	2	4	6	8	1	3	5	7	9

- Add up all sixteen digits in the modified number.
- Check that this total is a multiple of ten.

This works because one of the digits in the credit card number is chosen to ensure that a multiple of ten results from this procedure. This digit is called the 'check digit'.

(a) What is the missing digit (#) in 4920 1641 01#4 1711? [2]

A common mistake made when copying down numbers is to switch two adjacent digits (e.g. 24 miscopied as 42). The check digit system will detect single errors of this kind, except for one pair of digits.

(b) Which pair of digits in a card number will not be detected if they are switched? [1]

Card numbers are often read out as pairs of two-digit numbers; in the above example, "forty-nine, twenty, sixteen" etc. Some of these can be misheard, and in English this is particularly common for fourteen two-digit numbers:

13 heard as 30, or 30 heard as 13, and similarly 14 can be confused with 40, 15 with 50, 16 with 60, 17 with 70, 18 with 80, and 19 with 90.

(c) How many of these fourteen mistakes will be detected, if just one of them occurs when the sixteen-digit number of a card is read? [2]

(d) In how many of the sixteen positions could the check digit be located? (Assume nothing other than the information given.) [1]

(e) Four digits of a card number were blanked out "for security reasons".

4579 3991 #### 2607

How many possible valid card numbers could this represent? [2]

(f) A national identity register using sixteen-digit numbers for individuals was proposed, and it was suggested that the numbers should be allocated so that they could not be confused with a credit card number.

Specify a simple way to achieve this (without restricting the range of available credit card numbers). [2]

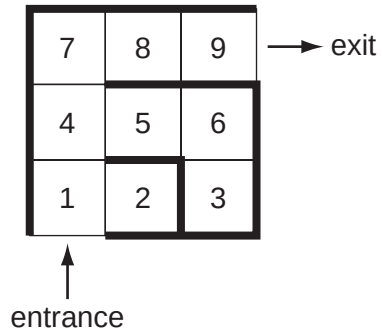
3 *Study the information below and answer the questions. Show your working.*

A games designer is trying to create rectangular mazes made up of identically-sized square rooms, and needs to quantify how complicated they are. In order to do this, he defines the longest path of a maze as the number of rooms that a participant would visit in completing the maze, if she were to take a wrong door whenever faced with a choice. Rooms visited more than once are counted for each visit.

The rooms all consist of four walls with closed doors in one or more of them. As a result the participant cannot know whether she has reached the exit until she opens the 'exit' door. The designer assumes that the participant marks each door that she tries, and will not repeat any pathways that she has already fully explored.

Because the maze designer is considering mazes of different sizes, he decides to use the *complexity coefficient* of the maze as the measure of its difficulty, which he defines as the longest path divided by the size of the maze (the number of rooms it consists of).

(a) Show that the *complexity coefficient* of the following maze is 1.44 (rounded to 2 decimal places), and complete the path that the participant takes: 1, 2, 1, 4… [2]



Mazes must not have loops in them. A loop is where a pathway leads a participant back to a room already visited without retracing her steps.

The entrance and the exit must be in different rooms. A participant must be able to reach all rooms in the maze.

(b) Draw a different maze involving a 3 x 3 grid of rooms, which also has a *complexity coefficient* of 1.44 and which has its entrance and exit in the same places as the maze above. [1]

(c) **(i)** The maximum *complexity coefficient* for a 3 x 3 maze is 1.78. Draw an example. (The entrance and exit may be in different positions to those in the example above.) [2]

(ii) Find the maximum *complexity coefficient* for a 5 x 5 maze. [2]

The designer also wants to consider the shortest way out of a maze. He defines the *simplicity coefficient* as the shortest path from the entrance to the exit divided by the size of the maze. In considering this, the designer restricts his investigation to rectangular mazes whose entrances and exits are in the rooms at diagonally opposite corners of the maze.

(d) Consider rectangular (including square) mazes made up from no more than 18 rooms. What is the lowest *simplicity coefficient* for such mazes? Give an example of a maze with this *simplicity coefficient*. [3]

The designer thinks it will be useful to consider the difference between the longest and shortest paths for particular mazes. When considering this, he continues to restrict himself to mazes in which the entrance and exit are in rooms at diagonally opposite corners of the maze.

(e) (i) For a maze made up from no more than 18 rooms, what is the maximum difference between the longest and shortest paths? Draw an example. [2]

(ii) For a 16 x 21 maze, what is the maximum difference between the longest and shortest paths? You are not required to draw an example. [3]

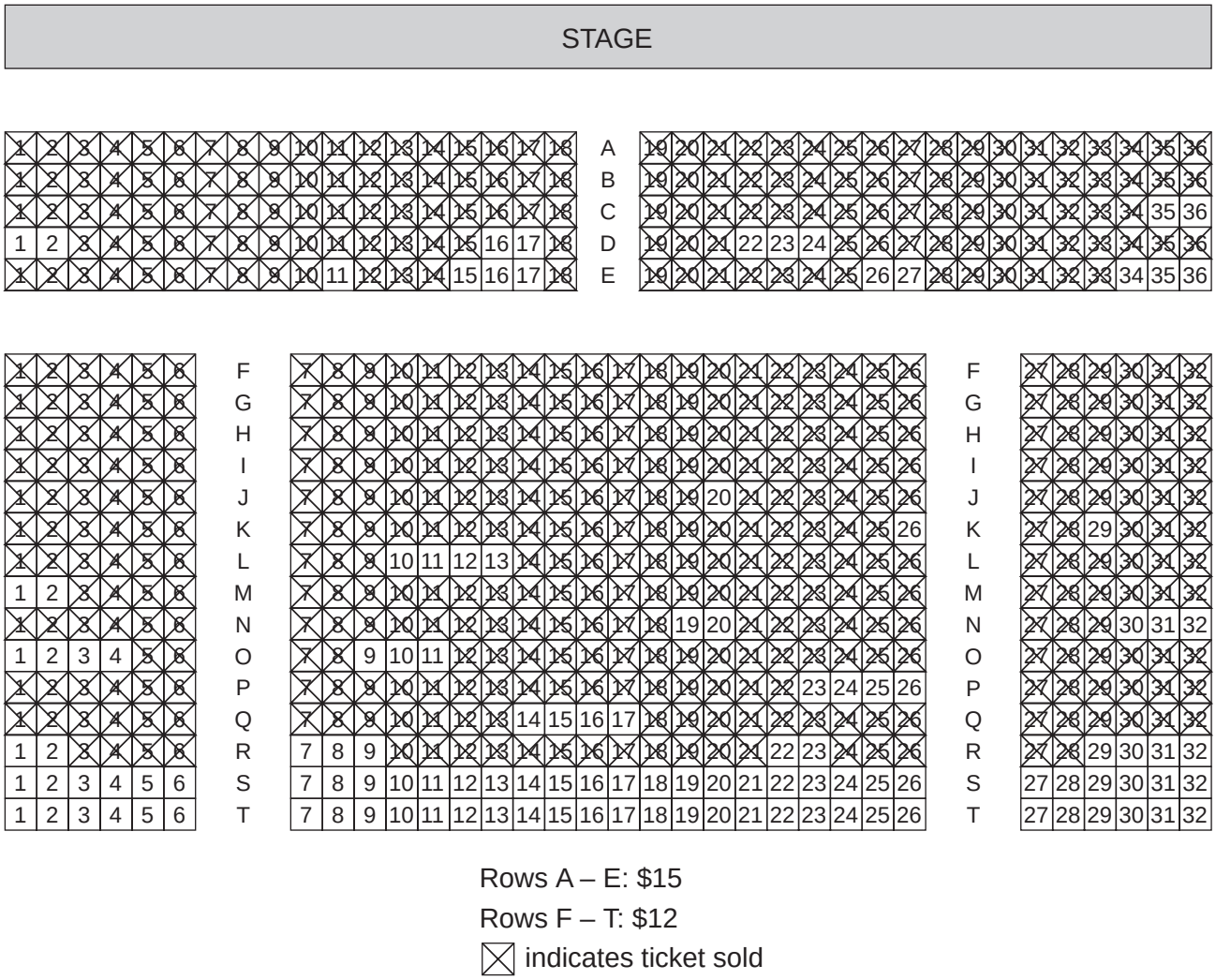
[Turn over

4 *Study the information below and answer the questions. Show your working.*

The Rodstein Musical Society is staging its production of North Atlantic at the Carousel Theatre each night this week from Monday to Saturday, with an extra performance tomorrow afternoon (Saturday). Tickets are priced at $15 and $12, except Monday when all tickets were $10.

All the tickets for tonight (Friday) and tomorrow night have been sold already, but there are still some available for tomorrow afternoon.

This is the theatre's seating plan for tomorrow afternoon's performance.



Rows A – E: $15

Rows F – T: $12

☒ indicates ticket sold

Two $12 tickets have just been sold to a customer, who chose two seats next to each other, as far forward as possible. They are about to be marked off as sold on the seating plan.

(a) (i) In which row are the two seats that have just been sold? [1]

(ii) How many $15 tickets have been sold so far for tomorrow afternoon's performance? [1]

(iii) How many $12 tickets have now been sold for tomorrow afternoon's performance? [1]

This is a summary of the sales of the tickets and programme booklets from the performances earlier this week.

	Number of tickets sold			*Total Ticket Sales*	*Programme Sales*
	$15	*$12*	*$10*		
Monday	–	–	344	$3440	$279
Tuesday	102	273	–		$384
Wednesday	115		–	$5433	$351
Thursday	136	368	–	$6456	$438

Two figures have been left out.

(b) (i) What was the income from total ticket sales for Tuesday's performance? [1]

(ii) How many $12 tickets were sold for Wednesday's performance? [2]

The Society's funds will receive all the profit from this week's performances of North Atlantic. Its profit is obtained by subtracting the hire of the theatre and all other costs (except those related to programme booklets) from the income from total ticket sales.

The selling of programme booklets at the performances is organised and run as a separate venture, with all the profit to be donated to a local youth club.

The cost of hiring the theatre is $1500 per **day**, plus $2 per ticket sold.

(c) (i) If no more tickets are sold for tomorrow afternoon's performance, what will be the total cost of hiring the theatre this week? [3]

(ii) What is the minimum profit the Society will make from this week's production when the other costs are $4360? [3]

Sales of programme booklets, at $3 each, have been very disappointing. 3000 were printed at a cost of $1200. $2470 was raised for the youth club last year, and this year's target is to beat this figure.

At an emergency meeting of the Programme Committee this morning, it was acknowledged that if sales do not improve, the amount raised will fall some way short of the target. It was decided to reduce the price, and that the new price should be the minimum multiple of $0.10 that will achieve the target if at least half of the remaining programme booklets are sold.

(d) What price did the Programme Committee set? [3]

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