A level Thinking Skills 9694
Unit 7: Data analysis, models and investigations

Recommended Prior Knowledge
Students need to have an understanding and proficiency in the skills and aptitudes of either O level Mathematics or IGCSE Mathematics.

Context
This unit involves the study of more advanced skills for problem solving. It builds on the skills which were introduced and developed in units 3 and 6. It prepares students for Questions 3 and 4 of Paper 3. As with Unit 6, although the skills are completely independent of the critical thinking units (1, 2, 4, 5 and 8), the problems which provide the raw material for the data analysis and interpretation may be useful in preparing for the critical reasoning needed in Question 1 of Paper 4.

Outline
The increased complexity of problems requires the development of mathematical methods for dealing with them, and this unit focuses on appropriate tools for modelling more complex situations, and dealing with more complex data. The section on investigations focuses on more open-ended problems, where what constitutes a solution is not prescribed.

Topic	Learning outcomes	Suggested Teaching activities	Learning resources
1	Developing a model	Initial example of how the use of an appropriate model can radically simplify the problem – the Konigsberg Bridge problem (see learning resources). The main options for modelling problems– when considering two dependent variables… graphs and algebra. For example – investment options problems. *You are offered two different schemes for investing your money. The Alpha Scheme offers to increase you money by 1% every year. The Omega Scheme offers to increase by $10 a year. If you start with $100, how many years would you have to invest money in order for the*	**Butterworth and Thwaites – Chapters 38, 39, 40, 41** **Polya – section on “Figures” (Pg 103-108)** gives advice on modelling geometric situations. **Polya – section on “setting up equations” (pages 174-7)** discusses the needs and pitfalls of modelling problems with algebra. http://mathforum.org/isaac/problems/bridges1.html is a good introduction to networks, with the Konigsberg Bridge problem explained and solved. http://nrich.maths.org/public/viewer.php?obj_id=2327&part=index&refpage= allows students to investigate the problem themselves. There are resources on The Route Inspection problem at: http://www.orsoc.org.uk/learn/16_plus_learning/16_plus_materials_route.htm

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		Alpha Scheme to be more lucrative than the Omega Scheme? When do graphs work? When considering the change in a certain quantity (or more generally, the relationship between two quantities). They rely on the ability to extrapolate and interpolate visually. When faced with more than two variables, or with a need for greater accuracy, algebra is needed. In the investment question above – *if t is time, and i is the total worth of the investment, the Alpha Scheme can be modelled as i =1.1^t x 100, and the Omega Scheme as i =100 + 10t. The investments yield the same when* *1.1^t x 100 = 100 + 10t.* *This is a difficult equation to solve!* When considering a three-dimensional problem - plans & elevations, nets and isometric drawings can all enable the problem to be modelled. Example 1 – *a spider wants to walk from one corner of a solid cube to the diagonally opposite corner – which route will be the shortest?* ***Solution – draw the net of a cube, and draw a straight line****.* Example 2 – *how many mooring buoys can be safely anchored to the bottom of a stretch of river 100m long, if the tide goes up and down 10m (at most) and boats (which may be 10m long) may swing back and forward, sometimes unpredictably?* ***Solution – this requires drawing an elevation of what a buoy would look like at high tide (vertical anchor chain) and low tide, and then a plan to see how far apart the buoys***	http://nrich.maths.org/public/leg.php?ct=0&cl=0&dl%5B0%5D=1&dl%5B1%5D=1&dl%5B2%5D=1&group_id=26&cldcmpid=&code=-172 http://www.purecoder.net/sixthworksheets/Route%20inspection.pdf The level of analysis to solve the Route Inspection problem is not needed for the Thinking Skills exam, but it is a good example of how an appropriate model can make the solution of a problem much easier.

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		could be to ensure that 10metre long swinging boats don't touch. Example – *a cube has six identical cubes stuck to its six faces. What is the greatest number of faces that can be seen at any one time on this new shape?* ***Solution – an isometric drawing of this shape helps to answer this visualisation question.*** When considering a series of interconnected events or positions networks help to focus on the important relationships. Example - *what is the shortest time in which a postman can walk down all the streets in a village and return to the start?* ***This is called the Route Inspection problem or the Chinese Postman problem, and there are many descriptions of it on the Internet (see Learning resources).***	
2	Carry out an investigation	Carrying out an investigation requires appropriate representation of the information being investigated (a good model), careful appreciation of any restrictions, and a continual focus on what the intended endpoint is. It requires the unification of the other sections of this unit, and a course could begin with a posed investigation, which was then tackled at the end.	**Butterworth and Thwaites – Chapter 42** Suitable vehicles for practising investigations are "Simplex problems" (which involve a complicated mathematical algorithm with a restricted domain of application, which candidates should ignore). Some examples and discussion of such problems can be found at : **http://www.math.tamu.edu/~jwhitfld/TQG_AlgII/2006-2007/LinearProgramming.pdf** and **http://math.uww.edu/~mcfarlat/simplex1.htm**
3	Analyse complex data and draw conclusions	One way of introducing the complexities of using large data sets is by asking students to consider what hypotheses could be supported by a broad	**Butterworth and Thwaites – Chapter 43** For large data sets, see

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		collection of data, such as those mentioned in the learning resources column. All that is needed is the column headings, for possible hypotheses to be suggested. Complex data will not necessarily mean large sets of data (particularly in the context of the exam), and students are not expected to be expert statisticians. Students should have experience in interrogating data in a number of forms – see resources. The mathematical skills required can be broken down into (a) how data is displayed – tables, graphs (b) pitfalls of summarising data (c) mathematical tools for summarising data and (d) drawing conclusions from data.	**http://www.stats4schools.gov.uk/large_datasets/default.asp** .