A level Thinking Skills 9694
Unit 6: Problem Solving – Advanced

Recommended Prior Knowledge
Students need to have an understanding and proficiency in the skills and aptitudes of either O level Mathematics or IGCSE Mathematics.

Context
This unit develops the problem-solving skills introduced in Unit 3, and involves their application to more advanced problems. It prepares students for the Problem Solving questions in Paper 3 of the A Level exam. Unlike the other problem-solving units, it is not completely independent of the critical thinking units (1, 2, 4, 5 and 8), since the problems relating to data selection and interpretation provide a useful basis for the development of critical reasoning needed in Question 1 of Paper 4 [dealt with Units 7 and 8].

Outline
Paper 3 tests similar skills to Paper 1, yet students will need to apply mathematical skills beyond those of junior/ elementary school (though not beyond IGCSE). Students will need to be able to apply: basic probability (including the combination of independent and non-independent events); linear and simultaneous equations to express and solve problems; integer powers; tree and decision diagrams; calculations of means, modes and medians; Venn and Carroll diagrams.

Topic	Learning outcomes	Suggested Teaching activities	Learning resources
1	Data selection and processing	Stubbornness and inventiveness are equally important parts of the intellectual toolkit required for problem-solving. Students should be given as much chance as possible to tackle problems (such as those on the puzzle website mentioned in the learning resources) in groups of 2 or 3 and report their findings back to the class.	For a selection of logical problems which involve careful reading and analysis, try www.folj.com/puzzles . These are graded according to difficulty.
2	Finding methods and procedures	The Monty Hall problem (the game show problem with the goats and the car and the doors) is a good way at introducing some of the complexities of probabilistic reasoning. See the Learning Resources column for details. A second problem which demonstrates the value of	**Polya – Part II How to Solve it : a dialogue** gives a set of general questions to be considered when tackling problems of any kind. **Polya – section on "variation of the problem" (pages 209-214), section on "wisdom of proverbs" (pages 221-5) and section on "working backwards" (pages 225-232)** give general advice on tackling problems the method for whose solution is unclear.

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		exhaustively tabulating the various options available in a problem is the 5 Hats problem. See the Learning Resources column for details.	There is also a variety of problems (not all of which are mathematical) in **Part IV Problems, hints, solutions**. A comprehensive overview of methods which can be used for problem-solving is available at http://en.wikipedia.org/wiki/Problem-solving . In particular see the section called "6. Problem-solving techniques". The Monty Hall problem is described on numerous sites on the Internet – http://en.wikipedia.org/wiki/Monty_Hall_problem for example. The 5 Hat problem, and a number of variations of it are described at http://www.everything2.com/index.pl?node_id=878489&lastnode_id=0. The problem is introduced as a simplified game in "Why do buses come in threes?" chapter 15.
3	Identifying relationships and hypotheses	This extends the skills introduced in Unit 3 to more complex and more real-life situations.	Useful websites for considering what hypotheses can be drawn from data include http://www.statistics.gov.uk/glance/ and http://www.fedstats.gov/cgi-bin/A2Z.cgi which contain a mass of graphs, many of which can be used to stimulate productive discussion about possible hypotheses.
4	Analysing and modelling	This extends the skills introduced in Unit 3 to more complex and more real-life situations. A good problem to engage students in the need for a good model is to ask groups of students to predict the population of their town in the year 2020, and justify their prediction. This can prompt discussion of the appropriateness of linear models and extrapolation. In order to refresh memories of the usefulness and of the skills required for solving simultaneous equations, try the following activity: students write down the first 5 terms of linear sequence, and then pair up and investigate whose sequence will give higher values as the sequences progress. For instance the sequence 6, 10, 14, 18, 22… overtakes the	A collection of questions about modelling data with a variety of different functions is to be found at : http://www.ncpublicschools.org/docs/accountability/testing/eoc/Algebra2/Samples/Fall2003AlgebraIIFormWS4.pdf

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		sequence 15, 18, 21, 24, 27 … at after the 10th term. This can be proved by a table, a graph, or by algebraic means. Students can guess when the sequences cross first (in order to inject some competitions into the process).	