AS/A level Thinking Skills 9694
Unit 3: Problem Solving

Recommended Prior Knowledge
Students need to have an understanding and proficiency in junior school Mathematics: in particular, the interpretation of graphs, the careful use of appropriate units (including compound units such as speed) and the ability to find fractions/percentages of a quantity.

Context
The unit focuses on the skills needed to solve quantitative problems, in which students must interrogate information which is given to them, in order to answer a particular question. It is independent of the critical thinking units (1, 2, 4, 5 & 8). The skills which are introduced in this unit will be developed in unit 6 (for which it is a prerequisite). It is expected that students will complete units 3 and 6 before studying unit 7, although they are not prerequisites, strictly speaking. This unit prepares students for the Problem Solving multiple-choice questions in Paper 1 of the AS exam.

Outline
This unit focuses on choosing the appropriate data and the appropriate method when attempting to solve a problem. This requires sensitivity to the information given in the light of the answer that is sought. It also requires an array of mathematical tools for simplifying and processing data – the most sophisticated being the choice of how to model the structure of a problem efficiently.

Topic	Learning outcomes	Suggested Teaching activities	Learning resources
1	Data selection and processing	Initial focus on paying attention to all the information given in a problem – a good way of drumming this home is a "decoy skills test" which begins with the instruction "read the whole paper first". The final question on the paper (which ought to be long enough to cover two sides, so that the final question is not immediately visible) says "ignore all the questions up to this point, put your pen down on your desk and wait silently". The intervening questions can become increasingly ridiculous and vocal (hopefully prompting students to see through the trick	**Butterworth and Thwaites** – Chapters 29-32 For a selection of logical problems which involve careful reading and analysis, try www.folj.com/puzzles . These are graded according to difficulty.

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		themselves). Students will be more wary of instructions afterwards. Focus on instructions developed by consideration of an unlabelled table, and inviting students to add titles/labels in order to give it context. Inviting students to consider labelled diagrams (for instance a table of departure times and journey lengths), and create questions themselves.	See June 2006 Thinking Skills paper 1 (8436/01) – questions 12-15, 25
2	Finding methods and procedures	"Simplifying the numbers" method for finding the appropriate calculation – if students are struggling to decide upon what operations to undertake in a problem, replacing large or "hard" numbers with easy counting numbers can reveal the appropriate pathway. Trial and improvement – the structured laying out of a table can be practised using simple mathematical problems such as "three children, who are each born four years apart, calculate that their ages add up to the same as their mother, who gave birth to the eldest when she was 20. How old is the mother when this happens?" How fallible our sense of structure is, is demonstrated by the "Missing Dollar" puzzle : *three diners pay for their meal, which costs $25 altogether. Each of the diners pays with $10 note. When they get their change ($5 in total), they decide to each take $1, and give the waiter $2 as a tip. The problem – they each paid $9 (their*	**Butterworth and Thwaites – Chapters 33, 34 and 35** **Polya – Part II How to Solve it : a dialogue** gives a set of general questions to be considered when tackling problems of any kind. **Polya – section on "variation of the problem" (pages 209-214)** gives general advice on tackling problems the method for whose solution is unclear. See June 2006 Thinking Skills paper 1 (8436/01) – question 11, 22, 23

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		original $10, with $1 returned) and gave the waiter $2 … but 27+2 = $29 not $30. Where did the missing dollar go? This problem can be resolved by replacing the numbers with less confusing numbers (try $13 for the meal in total, the diners still paying $10 each, and then offering to give the waiter a $2 tip after being given their change). Venn Diagrams – as a tool for analysing data which falls into a number of overlapping categories. Compound units – students need to feel confident in moving between units. DST triangles are a good way of doing this. A good "challenge question" to encourage complex conversion is – "would world-class 100 metre runners break the speed limit in the UK (30mph)?"	Practice at using Venn diagrams - http://www.shodor.org/interactivate/activities/VennDiagrams Light-hearted Venn diagrams for discussion – What is the author of these trying to say ? www.brunching.com/diagrams.html For an introduction to Venn diagrams and an online test, try www.philosophy.hku.hk/think/venn/tute1.php See June 2006 Thinking Skills paper 1 (8436/01) – question 3 See June 2006 Thinking Skills paper 1 (8436/01) – question 35
3	Identifying relationships and hypotheses	Using scatter diagrams – a way to introduce this is to give students graphs showing the three types of scatter diagram (positive, negative and no correlation) and ask them to label the axes with appropriate quantities and units. It's a good idea to work through a real-life example or two from science – e.g. Louis Pasteur and his flasks and the microbial growth patterns he observed within them. Alternatively get students to bring in / think up examples from their other subjects (especially those doing sciences).	**Butterworth and Thwaites** – Chapter 36 Discussion of correlation and causation could be prompted by the numerous examples on the website - http://jonathan.mueller.faculty.noctrl.edu/100/correlation_or_causation.htm
4	Analyzing and modelling	Graphical modelling – drawing distance/time graphs which represent walking across thee	**Butterworth and Thwaites** – Chapters 37 & 38

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		classroom in different ways [at a constant speed, stopping, accelerating, walking backwards]. Similarly speed/time graphs. 2D modelling of 3D objects – a way to develop this skill is to construct shapes out of Multilink cubes and ask students to draw front, side and top elevation of them. Then pass this triptych onto another class-member who must reconstruct the original 3D shape. Once the different categories of question have been covered, students can work in small groups on a series of questions and decide (quickly) which methods to use to tackle them; they can answer the questions and then report back to the class commenting on the effectiveness of the methods used. The teamwork approach can be varied so that, for example, different students are designated 'experts' in a particular technique, e.g. 'Analysing and Modelling', and questions distributed accordingly. To increase speed, specialist groups can be created; the whole class can then work against the clock by farming out questions to the appropriate people. People then swap roles and try and beat the previous time taken.	A resource for teachers on interpreting behaviour using graphs is to be found at www.clexchange.org/ftp/documents/Math/MA2000-10GraphingSkills.pdf#search=%22water%20bathtub%20graphs%22 See June 2006 Thinking Skills paper 1 (8436/01) – question 1 See June 2006 Thinking Skills paper 1 (8436/01) – question 2, 4, 43 There are a number of great games which require problem-solving skills, and which require players to model the logic of the game, represent it and often work back from a backwards solution. Highlights include the telescope game [http://www.dyson.co.uk/about/games/telescope.asp], Theseus and the Minotaur [http://www.logicmazes.com/theseus.html] and the cube logic game [http://www.microgames.info/game_info.php?products_id=205].