Section 3 of Unit 03 (Pure Mathematics)

Trigonometry

Recommended Prior Knowledge
Students must have covered the work on Trigonometry from P1.

Context
This is a small Section, but is one that does not depend upon other material. It corresponds to only a small part of the assessment. Most students find the topic of trigonometry difficult to comprehend, but it is essential that this work is completed before the work on integration.

Outline
The Section extends the work from P1 and introduces the three functions; secant, cosecant and cotangent. Graphs, equations and identities are considered using these functions. The various addition formulae are considered in detail and the work is extended by deducing and using the formulae for "double angles". The Section concludes by looking at the uses of expressing $a\sin x + b\cos x$ in the form $R\sin(x+\alpha)$ etc.

Topic	Learning Outcomes	Suggested Teaching Activities	Resources	On-Line Resources
3	**Trigonometry** • Understand the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude.	Discuss the definition of the secant, cosecant and cotangent functions and encourage the students to draw the graphs of these functions for themselves. It is worth having OHP slides showing the pairing of sine and cosecant, cosine and secant and tangent and cotangent on the same graphs. Use the properties of the graphs to solve simple equations of the form sec x = k etc. Discuss the turning points of the graphs of y=sec$x$ and y=cosec$x$.	OHP slides showing sine and cosecant on the same graph etc.	

	• Use trigonometric identities for the simplification and exact evaluation of expressions and in the course of solving equations, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of: • $\sec^2\theta \equiv 1+\tan^2\theta$ and $\cos ec^2\theta = 1+\cot^2\theta$, • the expansions of $\sin(A\pm B), \cos(A\pm B), \tan(A\pm B)$, • the formulae for sin2*A, cos2*A, tan2*A*.	Use the equation $\sin^2 x+\cos^2 x = 1$ to deduce that $\sec^2 x = 1+\tan^2 x$ and $\cos ec^2 x = 1+\cot^2 x$. Use these identities to solve equations in sec*x* and tan*x* (for example $\sec^2 x = 2\tan x$) and in cosec*x* and cot (for example $3\cos ec^2 x = 10\cot x$). Depending on the ability of the class and the time available, either prove or give the double angle formulae for $\sin(A\pm B)$, for $\cos(A\pm B)$ and for $\tan(A\pm B)$. Encourage the students to deduce expressions for sin2*A*, cos2*A* and tan2*A*. Encourage students to deduce the three different forms for *cos2x.* Discuss the different types of equations that can be solved using double angle formulae; such as:- • $\sin 2x = \cos x$, • $\cos 2x = 2\cos x + 1$, • $\cos 2x = 3 - 2\sin x$, • $\tan 2x = 3\tan x$, • $5\sin x\cos x = 1$. As an example of proving an identity, it is worth letting the students prove results for sin3*A*, cos3*A*, sin4*A* and cos4*A*. Students generally experience considerable difficulty with proving identities and will need a lot of practice with as many different examples as possible. Use compound angle formulae to show that $\sin\left(\frac{\pi}{2}-x\right) = \cos x$, sin(π−*x*) =sin*x*, tan$\left(\frac{\pi}{2}-x\right)$ =cot*x* etc and reconcile the results with the graphs of *y*=sin*x*, *y*=cos*x*, *y*=tan*x*, *y*=cot*x* etc.	OHP slides showing the full proof of the identities for sin3*A* and for cos4*A* etc.	

	• Show familiarity in the use of the expressions of $a\sin\vartheta + b\cos\theta$ in the forms $R\sin(\theta \pm \alpha)$ and $R\cos(\theta \mp \alpha)$.	Suggest that the students draw accurately for homework the graph of *y*= $a\sin\vartheta + b\cos\theta$, for a given a and *b* and use this graph as an introduction to the solution of the equation $a\sin\vartheta + b\cos\theta$ =k (The students should be able to appreciate that this equation cannot be solved by the use of any of the methods so far considered). Show the students how $a\sin\vartheta + b\cos\theta$ can be expressed as either $R\sin(\theta + \alpha)$ or as $R\cos(\theta - \alpha)$ and encourage students to proceed with the solution of the equation $a\sin\vartheta + b\cos\theta$ =k General discussion can now follow on finding either the maximum or minimum values of f(*θ*) where f(*θ*)= $a\sin\vartheta + b\cos\theta$. Students should be encouraged to find the stationary values by calculus and then to be directed to the same values (±*R*). Again, students will need plenty of practice solving equations of the form $a\sin\vartheta + b\cos\theta$ =k and in finding the maximum and minimum values of f(*θ*)= $a\sin\vartheta + b\cos\theta$ (or use range and domain for revision).	OHP slide showing an accurate graph of for example, y=3sinθ+4cosθ.	