UNIT 5 Calculus (P2)

Recommended Prior Knowledge. The basic properties of the exponential and logarithmic functions, particularly the differentials, must have been covered in Unit 4 (Algebra).
Context. This Unit extends the calculus studied in Unit 2 (Calculus and Coordinate Geometry). Because of the importance of a good understanding of calculus, this Unit has been placed before Unit 6 (Trigonometry). It is suggested that the work on integration, using trigonometric formulae, is left until after Unit 6 has been completed.

Outline. The work covered in Unit 2 (Calculus and Coordinate Geometry) is continued, but extends to the differentials of the three basic trigonometric functions and the exponential and logarithmic function. The differential of the product and quotient of two functions is studied, along with the first derivative of functions that are expressed either parametrically or implicitly. Integration as the reverse of differentiation is applied to the derivatives of the three basic trigonometric functions, to the derivatives of the exponential and logarithmic function and to the derivatives of composite functions as appropriate. Techniques of integration using the trigonometric formulae from Unit 5 should be left over until after Unit 6 (Trigonometry) has been completed. This Unit finishes by looking at the use of the trapezium rule for numerical integration.

Topic	Learning Outcomes	Suggested Teaching Activities	Resources	On-Line Resources
4	**Differentiation** • Use the derivatives of e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$, together with constant multiples, sums, differences, and composites.	The basic results for the derivatives of the three trigonometric functions can either be stated or verified by calculating gradients of tangents on accurate graphs (it will be necessary to define the secant function at this point if Unit 6 has not been completed). The need to emphasise to students that the results are correct only when measuring the angle in radians is essential. OHP graphs showing these basic results are particularly useful. It is expected that students will have met the derivatives of the exponential and logarithmic functions from the work covered in Unit 4. There is then the need for a lot of practice using the derivatives of composites of these functions.	OHP graphs showing the derivatives of the basic trigonometric functions.	www.bbc.co.uk/education/asguru/maths →pure →Differentiation →Differentiation of e^x and $\ln x$.
	• Differentiate products and quotients.	Extend the work to stating the formulae for the derivatives of products and quotients of two basic functions. It is worth	OHP showing all the derivatives from	

		suggesting that brighter students look at the proofs from first principles found in any A Level text-book, All the work on differentiation in Unit 2 (from P1) can now be revised using different functions. e.g. finding stationary points, equations of tangents and normals for curves of the type $y = x^2 e^{2x}$, y=$x \ln x$, y=$\dfrac{ax + b}{cx + d}$, y=$x$sin$x$ etc.	P1 and P2.
	• Find and use the first derivative of a function which is defined parametrically.	The idea of expressing a function by means of two parametric equations will be a new one for most students. Sketching accurately (a good exercise for homework) curves such as " $x = 2t$+3, $y = t - 4$ " or "x = t^2, y = t − 2" or " x=2sint , y=2cost" is a worthwhile exercise, and it is recommended that such accurate graphs are available on OHPs. If time allows, questions using the principles from Unit 2 on coordinate geometry can be repeated with curves expressed in parametric form. Students should also be able to find the cartesian form of functions expressed simply in parametric coordinates. Discuss fully with students, by reference to different examples, the derivative of a function that is defined parametrically. Use derivatives to find stationary values and where appropriate the greatest value of x, the equations of tangents and normals and other problems using the coordinate geometry principles in Unit 2 of P1 (Students are not expected to work with the second derivative but should be able to obtain the general form of the equation of the tangent and normal in the form y=m(t)x+c(t) where t is the parameter).	OHP showing an accurate graph obtained by plotting with parametric coordinates.
	• Find and use the first derivative of a function which is defined implicitly.	Discuss with students the idea of "explicit" and "implicit" functions. Show students that the derivative, with respect to x, of say y^2 can be obtained by use of the chain rule. Discuss the ideas behind finding the derivative of, say, $x^2 + y^2 = 100$. Look specifically at the use of the product	

		rule for the derivative of, say, "*xy*",

Students will need plenty of practice working with the derivatives of all the functions encountered in this section. In particular, they should be able to use derivatives to find stationary values, the equations of tangents and normals and other problems using the coordinate geometry principles in Unit 2 of P1. | | |
| 5 | **Integration**

• Extend the idea of "reverse differentiation" to include the integration of e^{ax+b}, $\dfrac{1}{ax+b}$, $\sin(ax+b)$, $\cos(ax+b)$, $\sec^2(ax+b)$ (knowledge of the general method of integration by substitution is not required).

• Use trigonometrical relationships (such as double-angle formulae) to facilitate the integration of functions such as $\cos^2 x$. | Students should obtain for themselves, the basic integrals of e^x, $\dfrac{1}{x}$, sin*x*, cos*x* and sec²*x*. Discussion should then follow on finding the integrals of these functions when "*x*" is replaced by "*ax+b*". Students should realise that any rules formulated do not apply to functions other than linear expressions.

Discuss with students problems met in Unit 2 requiring integration i.e. finding the equation of a curve from a gradient, and finding area s and volumes of revolution, but now using functions new to this Unit.

Particular problems to investigate include finding the area under a curve of the type $y = a\sin x + b\cos x$, or $y = \dfrac{12}{2x-3}$, or $y = e^x + e^{-x}$ etc or finding the volume under curves of the type $y = \dfrac{4}{ax+b}$ or $y = \dfrac{2}{\cos(ax+b)}$.

Students should be made aware that certain functions can be integrated by using an appropriate trigonometric formula. This section must however be left until after the work on | OHP showing a full list of all the integrals in P1 and P2. | www.bbc.co.uk/education/asguru/maths →pure →integration of e*x* and ln*x*. |

		trigonometry in Unit 6 has been completed. It is appropriate to return to this part of Unit 5 at a later stage (revising at the same time).		
		Students should be shown that		
		• the integral of both $\sin^2 x$ and $\cos^2 x$ can be achieved by the appropriate use of the formula for $\cos 2x$		
		• the integral of $\sin x \cos x$ can be achieved by the use of the formula for $\sin 2x$		
		• the integral of $\tan^2 x$ (and hence the volume of rotation of $y = \tan x$) can be obtained from the formula linking sec²x and $\tan^2 x$.		
	• Use the trapezium rule to estimate the value of a definite integral, and use sketch graphs in simple cases to determine whether the trapezium rule gives an over-estimate or an under-estimate.	Some students will have met the idea of using a numerical method for finding the approximate area under a curve (maybe in physics with the distance travelled under a v-t graph or in other maths syllabuses for finding the area below a curve).		
		Discussion should commence with a problem such as $\int_0^2 \sqrt{2x^2 + 1}\,\mathrm{d}x$. Students should recognise that this cannot be achieved by normal methods of integration. They should be encouraged to sketch the graph of y= $\sqrt{2x^2 + 1}$ and to suggest methods for finding an approximate value of the integral (have ready an OHP slide showing an accurate graph of $y = \sqrt{2x^2 + 1}$).They should appreciate that by taking more and more trapezia, more accurate results can be obtained. Although formal proof of the formula for using the trapezium rule is not required, most groups will appreciate and be able to understand it.	OHP slide showing the accurate graph of $y = \sqrt{2x^2 + 1}$. This should also show the approximate answers for the area when 2,3,4 or more strips are considered.	
		Students will need practice at using the rule and should be aware, from appropriate sketch graphs, as to whether the	Other OHP slides showing cases of "over-estimates" or	

| | | approximate answer is an under-estimate or an over-estimate of the true result. This is an appropriate place to use the OHP slide for $y=\sqrt{2x^2+1}$ to show whether an answer is an over-estimate or an under-estimate. Have other examples ready for the students along with corresponding OHP slides. These should include looking at $\int_0^1 \sqrt{1-x^2}\,\mathrm{d}x$ and showing that the trapezium rule underestimates the exact value, and that a sequence of estimates obtained from increasing the number of strips of equal width (say n=1,2,4,8…) converges to ¼π. | "under-estimates". | |