UNIT 5 Probability

Recommended Prior Knowledge. Students will have encountered simple ideas of probability in previous courses, and may have covered many of the ideas covered in this Unit. It is however essential to provide students with as many varied examples as possible.

Context. This unit is independent of Units 1 to 3 (Pure mathematics) though should follow Unit 4 (Numerical Statistics). For schools where two teachers are teaching the AS course, there is no problem in running Units 4 and 5 alongside Units 1 to 3. It is however preferable for students to have encountered the use of the binomial expansion in Unit 1 (Algebra) and then to link this with the binomial distribution in Unit 5.

Outline. The Unit looks at the topic of probability from a simple level, introducing students to the idea of independent and exclusive events and using tree diagrams and other techniques to evaluate probabilities of outcomes from 2 or more events. It looks at discrete probability distributions in general and the binomial distribution in particular. It concludes by looking in detail at the normal distribution and uses this as an approximation to the binomial distribution for large *n*.

Topic	Learning Outcomes	Suggested Teaching Activities	Resources	On-Line Resources
3	**Probability** • Evaluate probabilities in simple cases by means of enumeration of equiprobable elementary events (e.g. for the total score when two fair dice are thrown), or by calculation using permutations or combinations. • Use addition and multiplication of probabilities, as appropriate, in simple cases.	General discussion of probability as the chance of an outcome occurring in a particular event. Looking at simple examples of equiprobable events (e.g. the total score when two fair dice are thrown, or the number of heads when three fair coins are tossed). Discuss situations arising from simple permutations or combinations (e.g. the probability of two particular letters being selected from a total of 5). Look at the situation of two (or more) independent events and draw tree diagrams to illustrate the probabilities of particular outcomes occurring; e.g. 3 soldiers firing independently at a target when the probability of each hitting the target is given – students should be able to calculate the probability of each possible outcome – 0,1,2, or 3 hits. They should realise that the probability of exactly 2 hitting is the sum of three products. Have an OHP	OHP showing a tree diagram for 3 soldiers firing at a target and calculating the probabilities of the different outcomes.	www.bbc.co.uk/education/asguru →Maths →Methods →Probability (covers random events, addition laws, conditional probability, independence)

	• Understand the meaning of exclusive and independent events. • Calculate and use conditional probabilities, e.g. situations that can be represented by means of a tree diagram.	available illustrating the 8 outcomes on a tree diagram along with the resulting probabilities of each, Extending this to throwing 4 fair coins will introduce the idea of the binomial coefficients used in combinations and act as an introduction later to the binomial distribution. These examples will serve as an introduction to the idea of "exclusive" and "independent" events. Develop the formulae p(A)p(B)=p(A and B) for independent events and p(A and B) = 0 for mutually exclusive events. Extend the discussion above to include situations where the events are not independent. A good example of this is the case of a golfer hitting a straight drive, where the probability when the day is windy is different to when there is no wind. A more complex problem involves a 3-set tennis match where one of the players is temperamental: the probability of his winning a set decreases after he has just lost a set – could be Borg and McEnroe! Have an OHP ready to show the probability of each player winning the match when the probabilities are known.	OHP showing the possible outcomes of a 3-set tennis match when one player is so temperamental that the probability of his winning a set depends on whether he has won or lost the preceding one.	
4	**Discrete Random variables** • Construct a probability distribution relating to a given situation involving a discrete random variable X, and calculate E(X) and Var (X).	Discuss (as revision) the idea of continuous and discrete data. Talk to students generally about discrete probability distributions, looking at a number of different distributions. Emphasise to students that two type of situations arise:- those in which the distribution is given, usually in the form of a table of X against P(X) and those in which the students must find the distribution for themselves. Students should be able to work with a given probability distribution and be prepared to find a missing entry (using ΣP(X) = 1) and to		www.bbc.co.uk/education/asguru →Maths →Methods →Probability Distributions (covers discrete

		estimate the expected value (or mean) and the variance or standard deviation using stated formulae. It is worth asking students to find the mean and variance of a distribution such as that obtained from • throwing a fair dice (uniform distribution), • the total score when the same dice is thrown twice, • the number of heads obtained when 3 coins are tossed Students need as much practice from different types of questions as possible.		random variables and expected value). →Maths →Statistics →Discrete random Variable → Binomial Distribution (These cover all the work on discrete random variables, and on the binomial distribution).
	• Use formulae for probabilities for the binomial distribution, and recognise practical situations where the binomial distribution is a suitable model (the notation B(n, p) is included).	General discussion of what is meant by a binomial distribution. Emphasise from the outset the need for the probability in each event to be constant; one event independent of each of the others. Encourage students to look in detail at one particular distribution; e.g. the distribution of the number of times a train is late over a period of 5 days, given that the probability of a train being late on any one day is 0.4 (this is a good place to introduce the notation B(5,0.4)). Encourage the students to find the probability distribution for themselves, but have the full distribution ready on an OHP. Use the answers to generalise the formula for probabilities for a general binomial distribution.	Prepare OHPs showing the probability distribution for X~B(5,0.4) and X~B(4,0.5).	
	• Use formulae for the expectation and variance of the binomial distribution.	Encourage students to find the expectation and variance for the binomial distribution just considered and verify the formulae for E(X)=np and Var(X)=np(1-p). Repeat the process with a different binomial distribution, say X~B(4,0.5) where X is the number of heads obtained when 4 fair coins are tossed simultaneously.		

5	**The Normal Distribution**			
	• Understand the use of a normal distribution to model a continuous random variable and use normal distribution tables.	Talk in general terms about continuous distributions, mentioning that P(X=k) = 0. Introduce the idea of a normal distribution, possibly by looking at a few different distributions e.g. the volume of fruit juice in a (supposedly) 1 litre bottle, the time for a large group of athletes to run a 100m race, the mass of eggs laid by a large group of hens over a period of time. Give the students the mean and standard deviation of each graph and use this occasion to introduce the notation of $N(\mu, \sigma^2)$. Encourage students to recognise for themselves the symmetry of the graphs of these distributions. Have ready an OHP showing the three distributions side by side and discuss the properties that all three graphs have in common; in particular that the same proportion (or percentage) of items are, say, exactly one standard deviation above the mean.	Prepare an OHP showing three different normal distributions each with a given mean and variance.	→Maths →Statistics →Normal Distribution This site covers all the work needed for this section.
	• Solve problems concerning a variable X, where X from $N(\mu, \sigma^2)$, including finding the value, given the values of x_1, μ, σ. of $P(X > x_1)$, or a related probability, finding a relationship between x_1, μ, and σ given the value of $P(X > x_1)$ or a related probability.	Introduce normal distribution tables and link this with the discussion above. Introduce the idea of a standardised variable (z) for a particular distribution, say X~N(125,400) and discuss with the students the methods of finding, such probabilities as P(X > 135), P(X > 120), P(X < 140), P(X < 110), P(115 < X <130). Discuss with students the fact that P(X=k)=0. Discuss with students the problem of working backwards from a given probability (or percentage, or proportion) to finding an unknown (say, μ or σ or a particular value of X). Able students should be able to cope with the problem of knowing two percentage points of a distribution and using simultaneous equations to find the mean and variance.	Have normal distribution tables ready for use.	
	• Recall conditions under which the			

	normal distribution can be used as an approximation to the binomial distribution (*n* large enough to ensure that *np* > 5 and *nq* > 5) and use this approximation, with a continuity correction, in solving problems.	Discuss with students the binomial distribution B(20,0.6). Let different students within the group evaluate the actual probabilities in this distribution. and deduce the accurate answer to the question "Find the probability of more than 14 successes". Have ready an OHP showing the calculated probabilities on a histogram. Students should recognise that the resulting shape is very much "normal" in shape. Shade the area on the normal curve for which *X*>14. Discuss with students the method for finding the probability of "more than 14 successes" if the distribution were "normal". Students should appreciate that the mean and variance should be the same as the mean and variance of B(20.0.6) and that it is necessary to use a continuity correction, evaluating *z* as $\dfrac{14.5 - 12}{\sqrt{4.8}}$. Students can now compare and discuss their answers. [It is a good teaching technique to let students go their own way without a "continuity correction", and then to let them find out for themselves the reason for the difference in the answers.] If time allows, let students repeat this exercise with a different binomial distribution.	OHP showing the probability of obtaining *X* successes form B(20.0.6) for all *X*. These should be drawn on a histogram.	