

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS General Certificate of Education Advanced Subsidiary Level

MATHEMATICS 9709/23

Paper 2 Pure Mathematics 2 (P2)

October/November 2012

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper

Graph Paper

List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of 3 printed pages and 1 blank page.



1 Solve the inequality $|x-2| \ge |x+5|$.

2 Use logarithms to solve the equation $5^x = 3^{2x-1}$, giving your answer correct to 3 significant figures. [4]

3 Solve the equation

$$2\cos 2\theta = 4\cos \theta - 3,$$

for
$$0^{\circ} \le \theta \le 180^{\circ}$$
. [4]

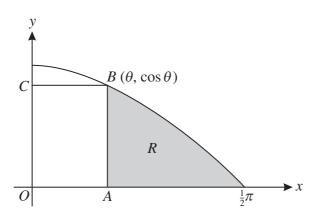
4 The parametric equations of a curve are

$$x = \ln(1 - 2t), \quad y = \frac{2}{t}, \quad \text{for } t < 0.$$

(i) Show that
$$\frac{dy}{dx} = \frac{1-2t}{t^2}$$
. [3]

(ii) Find the exact coordinates of the only point on the curve at which the gradient is 3. [3]

5



The diagram shows the curve $y = \cos x$, for $0 \le x \le \frac{1}{2}\pi$. A rectangle *OABC* is drawn, where *B* is the point on the curve with *x*-coordinate θ , and *A* and *C* are on the axes, as shown. The shaded region *R* is bounded by the curve and by the lines $x = \theta$ and y = 0.

(i) Find the area of R in terms of θ . [2]

(ii) The area of the rectangle OABC is equal to the area of R. Show that

$$\theta = \frac{1 - \sin \theta}{\cos \theta}.$$
 [1]

(iii) Use the iterative formula $\theta_{n+1} = \frac{1 - \sin \theta_n}{\cos \theta_n}$, with initial value $\theta_1 = 0.5$, to determine the value of θ correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

[3]

(a) Use the trapezium rule with two intervals to estimate the value of 6

$$\int_0^1 \frac{1}{6+2e^x} dx,$$

giving your answer correct to 2 decimal places.

(b) Find
$$\int \frac{(e^x - 2)^2}{e^{2x}} dx$$
. [4]

- The polynomial $2x^3 4x^2 + ax + b$, where a and b are constants, is denoted by p(x). It is given 7 that when p(x) is divided by (x + 1) the remainder is 4, and that when p(x) is divided by (x - 3) the remainder is 12.
 - (i) Find the values of a and b. [5]
 - (ii) When a and b have these values, find the quotient and remainder when p(x) is divided by $(x^2 2)$.
- (i) By differentiating $\frac{1}{\cos \theta}$, show that if $y = \sec \theta$ then $\frac{dy}{d\theta} = \tan \theta \sec \theta$. 8 [3]
 - (ii) Hence show that

$$\frac{\mathrm{d}^2 y}{\mathrm{d}\theta^2} = a \sec^3 \theta + b \sec \theta,$$

giving the values of a and b.

[4]

(iii) Find the exact value of

$$\int_0^{\frac{1}{4}\pi} (1 + \tan^2 \theta - 3 \sec \theta \tan \theta) \, d\theta.$$
 [5]

[3]

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