UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2011 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/33 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2011 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

UNIVERSITY of CAMBRIDGE
International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	**Either** Obtain correct unsimplified version of x or x^2 term in expansion of $(2+x)^{-2}$ or $(1+\frac{1}{2}x)^{-2}$	M1	
	Correct first term 4 from correct work	B1	
	Obtain $-4x$	A1	
	Obtain $+\,3x^2$	A1	
	Or Differentiate and evaluate f(0) and f″(0) where f″$(x) = k(2+x)^{-3}$	M1	
	State correct first term 4	B1	
	Obtain $-4x$	A1	
	Obtain $+\,3x^2$	A1	[4]
2	Use correct quotient or product rule or equivalent	M1	
	Obtain $\frac{(1+e^{2x}).2e^{2x}-e^{2x}.2e^{2x}}{(1+e^{2x})^2}$ or equivalent	A1	
	Substitute $x = \ln 3$ into attempt at first derivative and show use of relevant logarithm property at least once in a correct context	M1	
	Confirm given answer $\frac{9}{50}$ legitimately	A1	[4]
3	**(i)** State or imply $R = 17$	B1	
	Use correct trigonometric formula to find α	M1	
	Obtain 61.93° with no errors seen	A1	[3]
	(ii) Evaluate $\cos^{-1}\frac{12}{R}$ (= 45.099)	M1	
	Obtain answer 107.0°	A1	
	Carry out correct method for second answer	M1	
	Obtain answer 16.8° and no others between 0° and 360°	A1	[4]

4	**(i)**	Separate variables and attempt integration on both sides	M1*	
		Obtain $2N^{0.5}$ on left-hand side or equivalent	A1	
		Obtain $-60e^{-0.02t}$ on right-hand side or equivalent	A1	
		Use 0 and 100 to evaluate a constant or as limits in a solution containing terms $aN^{0.5}$ and $be^{-0.02t}$	DM1*	
		Obtain $2N^{0.5} = -60e^{-0.02t} + 80$ or equivalent	A1	
		Conclude with $N = (40 - 30e^{-0.02t})^2$ or equivalent	A1	[6]
	(ii)	State number approaches 1600 or equivalent, following expression of form $(c + de^{-0.02t})^n$	B1√	[1]
5	**(i)**	**Either** Use integration by parts and reach an expression $kx^2 \ln x \pm n \int x^2 . \frac{1}{x} dx$	M1	
		Obtain $\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x \, dx$ or equivalent	A1	
		Obtain $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$	A1	
		Or Use Integration by parts and reach an expression $kx(x\ln x - x) \pm m \int x\ln x - x dx$	M1	
		Obtain $I = (x^2 \ln x - x^2) - I + \int x dx$	A1	
		Obtain $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$	A1	
		Substitute limits correctly and equate to 22, having integrated twice	DM1*	
		Rearrange and confirm given equation $a = \sqrt{\frac{87}{2\ln a - 1}}$	A1	[5]
	(ii)	Use iterative process correctly at least once	M1	
		Obtain final answer 5.86	A1	
		Show sufficient iterations to 4 d.p. to justify 5.86 or show a sign change in the interval (5.855, 5.865)	A1	
		$(6 \rightarrow 5.8030 \rightarrow 5.8795 \rightarrow 5.8491 \rightarrow 5.8611 \rightarrow 5.8564)$		[3]

6	(i)	Use correct method for finding modulus of their w^2 or w^3 or both	M1	
		Obtain $\|w^2\| = 2$ and $\|w^3\| = 2\sqrt{2}$ or equivalent	A1	
		Use correct method for finding argument of their w^2 or w^3 or both	M1	
		Obtain $\arg(w^2) = -\frac{1}{2}\pi$ or $\frac{3}{2}\pi$ and $\arg(w^3) = \frac{1}{4}\pi$	A1ft	[4]
	(ii)	Obtain centre $-\frac{1}{2}-\frac{1}{2}i$ (their w^2)	B1ft	
		Calculate the diameter or radius using $\|w-w^2\|$ w21 or right-angled triangle or cosine rule or equivalent	M1	
		Obtain radius $\frac{1}{2}\sqrt{10}$ or equivalent	A1	
		Obtain $\|z+\frac{1}{2}+\frac{1}{2}i\| = \frac{1}{2}\sqrt{10}$ or equivalent	A1ft	[4]
7	(i)	Substitute $x = \frac{1}{2}$ and equate to zero		
		or divide by $(2x-1)$, reach $\frac{a}{2}x^2 + kx + \ldots$and equate remainder to zero		
		or by inspection reach $\frac{a}{2}x^2 + bx + c$ and an equation in b/c		
		or by inspection reach $Ax^2 + Bx + a$ and an equation in A/B	M1	
		Obtain $a = 2$	A1	
		Attempt to find quadratic factor by division or inspection or equivalent	M1	
		Obtain $(2x-1)(x^2+2)$	A1cwo	[4]
	(ii)	State or imply form $\dfrac{A}{2x-1}+\dfrac{Bx+C}{x^2+2}$, following factors from part **(i)**	B1√	
		Use relevant method to find a constant	M1	
		Obtain $A = -4$, following factors from part **(i)**	A1√	
		Obtain $B = 2$	A1	
		Obtain $C = 5$	A1	

8	**(i)**	Differentiate y to obtain $3\sin^2 t\cos t - 3\cos^2 t\sin t$ o.e.	B1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dt}{dx}$	M1	
		Obtain given result $-3\sin t\cos t$	A1cwo	[3]
	(ii)	Identify parameter at origin as $t = \frac{3}{4}\pi$	B1	
		Use $t = \frac{3}{4}\pi$ to obtain $\frac{3}{2}$	B1	[2]
	(iii)	Rewrite equation as equation in one trig variable e.g. $\sin 2t = -\frac{2}{3}$, $9\sin^4 x - 9\sin^2 x + 1 = 0$, $\tan^2 x + 3\tan x + 1 = 0$	B1	
		Find at least one value of t from equation of form $\sin 2t = k$ o.e.	M1	
		Obtain 1.9	A1	
		Obtain 2.8 and no others	A1	[4]

9	(i)	Calculate scalar product of direction of l and normal to p	M1	
		Obtain $4 \times 2 + 3 \times (-2) + (-2) \times 1 = 0$ and conclude accordingly	A1	[2]
	(ii)	Substitute $(a, 1, 4)$ in equation of p and solve for a	M1	
		Obtain $a = 4$	A1	[2]
	(iii)	**Either** Attempt use of formula for perpendicular distance using $(a, 1, 4)$	M1	
		Obtain at least $\frac{2a-2+4-10}{\sqrt{4+4+1}} = 6$	A1	
		Obtain $a = 13$	A1	
		Attempt solution of $\frac{2a-8}{3} = -6$	M1	
		Obtain $a = -5$	A1	
		Or Form equation of parallel plane and substitute $(a, 1, 4)$	M1	
		Obtain $\frac{2a+2}{3} - \frac{10}{3} = 6$	A1	
		Obtain $a = 13$	A1	
		Solve $\frac{2a+2}{3} - \frac{10}{3} = -6$	M1	
		Obtain $a = -5$	A1	
		Or State a vector from a pt on the plane to $(a, 1, 4)$ *e.g.* $\begin{pmatrix} a-5 \\ 1 \\ 4 \end{pmatrix}$ or $\begin{pmatrix} a \\ 1 \\ -6 \end{pmatrix}$	B1	
		Calculate the component of this vector in the direction of the unit normal and equate to 6 : $\frac{1}{3}\begin{pmatrix} a-5 \\ 1 \\ 4 \end{pmatrix}.\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = 6$	M1	
		Obtain $a = 13$	A1	
		Solve $\frac{1}{3}\begin{pmatrix} a-5 \\ 1 \\ 4 \end{pmatrix}.\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = -6$	M1	
		Obtain $a = -5$	A1	

		Or		
		State or imply perpendicular line $\mathbf{r} = \begin{pmatrix} a \\ 1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$	B1	
		Substitute components for p and solve for μ	M1	
		Obtain $\mu = \dfrac{8-2a}{9}$	A1	
		Equate distance between $(a, 1, 4)$ and foot of perpendicular to ±6	M1	
		Obtain $\dfrac{3(8-2a)}{9} = \pm 6$ or equivalent and hence –5 and 13	A1	[5]
10	**(i)**	State or imply $\dfrac{du}{dx} = \sec^2 x$	B1	
		Express integrand in terms of u and du	M1	
		Integrate to obtain $\dfrac{u^{n+1}}{n+1}$ or equivalent	A1	
		Substitute correct limits correctly to confirm given result $\dfrac{1}{n+1}$	A1	[4]
	(ii)	**(a)** Use $\sec^2 x = 1 + \tan^2 x$ twice	M1	
		Obtain integrand $\tan^4 x + \tan^2 x$	A1	
		Apply result from part **(i)** to obtain $\frac{1}{3}$	A1	[3]
		Or Use $\sec^2 x = 1 + \tan^2 x$ and the substitution from **(i)**	M1	
		Obtain $\int u^2 \mathrm{d}u$	A1	
		Apply limits correctly and obtain $\frac{1}{3}$	A1	
		(b) Arrange, perhaps implied, integrand to $t^9 + t^7 + 4(t^7 + t^5) + t^5 + t^3$	B1	
		Attempt application of result from part **(i)** at least twice	M1	
		Obtain $\dfrac{1}{8} + \dfrac{4}{6} + \dfrac{1}{4}$ and hence $\frac{25}{24}$ or exact equivalent	A1	[3]