

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS General Certificate of Education Advanced Subsidiary Level

MATHEMATICS 9709/22

Paper 2 Pure Mathematics 2 (P2)

October/November 2009

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper

Graph Paper

List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

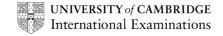
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of 3 printed pages and 1 blank page.



1 Solve the inequality |x + 3| > |2x|.

[4]

2 It is given that ln(y + 5) - ln y = 2 ln x. Express y in terms of x, in a form not involving logarithms.

[4]

3 (i) Use the trapezium rule with two intervals to estimate the value of

$$\int_0^{\frac{1}{3}\pi} \sec x \, \mathrm{d}x,$$

giving your answer correct to 2 decimal places.

[3]

- (ii) Using a sketch of the graph of $y = \sec x$ for $0 \le x \le \frac{1}{3}\pi$, explain whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part (i). [2]
- 4 The parametric equations of a curve are

$$x = 1 - e^{-t}, \quad y = e^{t} + e^{-t}.$$

(i) Show that
$$\frac{dy}{dx} = e^{2t} - 1$$
. [3]

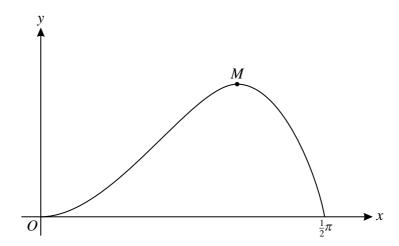
- (ii) Hence find the exact value of t at the point on the curve at which the gradient is 2. [2]
- 5 The polynomial $ax^3 + bx^2 5x + 2$, where a and b are constants, is denoted by p(x). It is given that (x+1) and (x-2) are factors of p(x).
 - (i) Find the values of a and b. [5]
 - (ii) When a and b have these values, find the other linear factor of p(x). [2]
- 6 (i) Express $3\cos x + 4\sin x$ in the form $R\cos(x \alpha)$, where R > 0 and $0^{\circ} < \alpha < 90^{\circ}$, stating the exact value of R and giving the value of α correct to 2 decimal places. [3]
 - (ii) Hence solve the equation

$$3\cos x + 4\sin x = 4.5,$$

giving all solutions in the interval $0^{\circ} < x < 360^{\circ}$.

[4]

7



The diagram shows the curve $y = x^2 \cos x$, for $0 \le x \le \frac{1}{2}\pi$, and its maximum point M.

(i) Show by differentiation that the x-coordinate of M satisfies the equation

$$\tan x = \frac{2}{x}.$$
 [4]

- (ii) Verify by calculation that this equation has a root (in radians) between 1 and 1.2. [2]
- (iii) Use the iterative formula $x_{n+1} = \tan^{-1}\left(\frac{2}{x_n}\right)$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

8 (a) Find the exact value of
$$\int_0^{\frac{1}{3}\pi} (\sin 2x + \sec^2 x) dx.$$
 [5]

(b) Show that
$$\int_{1}^{4} \left(\frac{1}{2x} + \frac{1}{x+1} \right) dx = \ln 5.$$
 [4]

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