MATHEMATICS

Paper 9709/01
Paper 1

General comments

The paper allowed the vast majority of candidates to show what they had learnt. There were however several questions which caused most candidates some difficulty (**Questions 5**, **7** and **9**) but allowed the more able candidates to produce work of a high quality. There were many excellent scripts and not so many scripts at the lower end. The standard of algebraic manipulation varied considerably from Centre to Centre and the level of algebra required in **Questions 7(i)** and **10(ii)** proved too much for many candidates. The standard of presentation was generally pleasing and the majority of candidates followed the rubric by showing their working in full.

Comments on specific questions

Question 1

The majority of candidates realised that the coefficient of x^2 came from the term ${}^6C_2\left(\frac{x}{2}\right)^4\left(\frac{2}{x}\right)^2$ but this usually followed after writing down the whole expansion. Only a small minority used the general term. It was pleasing that the binomial coefficient was rarely omitted. A significant number of candidates offered both the coefficients of x^2 and x^{-2} and many of these added these coefficients.

Answer: $3\frac{3}{4}$.

Question 2

There was a definite improvement in the way most candidates tackled this question on trigonometric identities and the majority coped with the algebra required in adding the two fractions and with recognising the need to use the identity $\sin^2 x + \cos^2 x = 1$. The most common error was to expand $(1+\sin x)^2$ as $1+\sin^2 x$ and surprisingly there were a large number of candidates who failed to proceed from $\frac{2+2\sin x}{\cos x(1+\sin x)}$ to the given answer.

Question 3

This proved to be a straightforward question with most candidates obtaining a correct value for the common difference and proceeding to use the formula for the sum of n terms to obtain a quadratic in n. The most common errors occurred through incorrect algebra in either forming or solving the quadratic equation.

Answer: 8.

Question 4

It was pleasing to see many correct solutions and a very good understanding of the concept of scalar product. Unfortunately sign errors were very common in part **(i)**, especially the error of expressing $\overrightarrow{PA}$ as $\overrightarrow{AP}$. Similarly in part **(ii)**, a large number of candidates used $\overrightarrow{AP}.\overrightarrow{PN}$ instead of $\overrightarrow{PA}.\overrightarrow{PN}$ to calculate angle *APN*. A similar number of candidates, having obtained a scalar product of −16, incorrectly assumed that the modulus of this was needed in order to evaluate the angle.

Answers: **(i)** −6**i** − 8**j** − 6**k**, 6**i** + 2**j** − 6**k**; **(ii)** 99.0º.

Question 5

This question was very poorly answered with only a small majority of candidates giving a fully correct answer. The restriction of 'where *a* and *b* are positive constants' meant that $a+b=10$ and $a-b=-2$, rather than the more widely found '$a+b=-2$' and '$a-b=10$'. Most candidates realised the need in part **(ii)** to make cos *x* the subject, but many of them only gave one answer. The sketch graphs in part **(iii)** were mixed with most realising that there was one complete cycle in the range 0° to 360°, but many candidates produced graphs that were very nearly 'V' or inverted 'V' shaped shapes.

Answers: **(i)** 4, 6; **(ii)** 48.2°, 311.8°.

Question 6

This proved to be a straightforward question that generally produced high marks. Part **(i)** presented a few problems with some candidates failing to realise the need to use $s = r\theta$ but it was pleasing that most coped comfortably with using radian measure. Apart from a small minority who failed to halve the angle found in part **(i)**, parts **(ii)** and **(iii)** generally produced correct answers.

Answers: **(i)** 1.8 radians; **(ii)** 6.30 cm; **(iii)** 9.00 cm².

Question 7

Part **(i)** caused lot of problems with at least a third of all attempts failing to recognise the need to obtain two simultaneous equations in *x* and *r*. Of the others, most attempted to make *r* the subject, but the algebra required in squaring $\left(\frac{40-2x}{\pi}\right)$ defeated at least a half of them. Part **(ii)** was well answered and it was pleasing to see that most of the candidates who were unable to answer part **(i)** realised that part **(ii)** was still accessible. Nearly all candidates realised the need to differentiate and to set the differential to 0.

Answer: **(ii)** 11.2.

Question 8

This proved to be a source of high marks and there were a large number of perfectly correct solutions. In part **(i)** most candidates differentiated correctly and used the formula $m_1m_2 = -1$ to find the gradient of the normal, though many weaker candidates failed to realise the need to express this as a number prior to finding the equation of a straight line. Parts **(ii)** and **(iii)** were usually correctly answered.

Answers: **(ii)** (−8, 6); **(iii)** 11.2.

Question 9

In part **(i)** a minority of solutions found the answer directly from making x the subject and finding the area by using $\int x \, dy$. Of those using $\int y \, dx$, failure to obtain the correct multiplying constant in the integration of $(3x+1)^{\frac{1}{2}}$ or to subtract the resulting area from the area of a rectangle (2) meant that there was only a small proportion of these candidates obtaining a correct answer. Similarly in part **(ii)**, many candidates either incorrectly attempted to find the volume about the y-axis, or failed to subtract the volume about the x-axis from the volume of a cylinder. There were very few correct answers for part **(iii)** with most of them coming from use of the formula for $\tan(A - B)$ (a P3 topic). Only a small minority recognised that the gradient of the tangent can be equated to $\tan\theta$, where θ is the angle made with the x-axis.

Answers: **(i)** $\frac{4}{9}$; **(ii)** 4.71; **(iii)** 19.4°.

Question 10

The general response to this question was mixed. Parts **(iii)** and **(iv)** were very well answered; parts **(i)** and **(ii)** much less so. In part **(i)**, most candidates managed to sketch the graph of $y = f(x)$ but many failed to realise that the graph of $y = f^{-1}(x)$ was the reflection of the graph of $y = f(x)$ in the line $y = x$. In part **(ii)**, nearly all candidates obtained a correct expression for $gf(x)$ but failed to cope with the algebra of simplifying $6(3x-2)-(3x-2)^2$ to obtain a correct quadratic. Those proceeding to differentiate to obtain a correct value of x at the stationary value and hence to prove the required result fared much better than those attempting to complete the square. A very large number of these candidates changed the sign of the quadratic but failed either to complete the square correctly or to reverse the sign at a later time. Part **(iii)** presented very few problems and it was pleasing that most candidates realised the need to use the answer to part **(iii)** in order to find the inverse of h. Several candidates however left the answer in terms of y instead of x.

Answers: **(ii)** $-9x^2 + 30x - 16$; **(iii)** $9-(x-3)^2$; **(iv)** $3+\sqrt{9-x}$.

MATHEMATICS

Paper 9709/02
Paper 2

General comments:

Candidates generally showed poor understanding of the basic rules and results of the calculus in **Questions 5**, **6**, and **8(ii)**, **(iii)**.

Many marks were lost by candidates using the correct methods but spoiling promising solutions by careless mistakes; often seen, for example, was the error '$2a + 26 = 0$, so $a = +13$' in **Question 2(i)**. Such errors jeopardise later parts of the working.

Where *exact* answers were requested, marks were often lost by use of *approximate* values, e.g. in **Question 4(i)** and the later stages of **Question 5**. Candidates are advised to read the questions more carefully.

Standards of neatness and presentation were very high. There was no evidence of candidates having too little time to complete the paper.

Candidates are strongly advised to prepare themselves by working through, and studying carefully, previous papers.

Comments on individual questions

Question 1

Almost everyone squared each side to obtain a quadratic equation (or inequality) in x. Others failed to proceed beyond $x = -3$ or $x = +1$. Although the question was generally successfully attempted, the final inequality was often incorrectly formed. Candidates are advised to check if the value $x = 0$ satisfies the original inequality; if it does, then it must belong to the final solution set.

Answer: $-3 < x < 1$.

Question 2

(i) Candidates invariably noted that $p(-2) = 0$, but solutions were often marked by a variety of sign errors.

(ii) A correct value for a was usually followed by a correct solution here, but many candidates unnecessarily solved the equation $p(x) = 0$.

Answers: **(i)** -13; **(ii)** $(x + 2)(2x + 1)(x - 3)$.

Question 3

Many candidates failed to attempt this question. Here $\ln y$ (not y itself) is linear in x, thus $\ln y = \ln A - x \ln b$, with $\ln A$ being the intecept on the vertical axis and $-\ln b$ being the gradient ($= -\frac{1}{4}$) of the line.

Answers: 3.67, 1.28.

Question 4

(i) The equation was often given as $\sin x \cos 60° + \cos x \sin 60° = 2 (\cos x \cos 60° + \sin x \sin 60°)$ or the right-hand side of the equation as $(2 \cos x \times 2 \cos 60° - 2 \sin x \times 2 \sin 60°)$. Many good solutions stopped short of a final answer.

(ii) Around half of all candidates derived an equation tan x = $3\sqrt{3}$ or tan x = 0, instead of the correct form tan x = $\frac{1}{3\sqrt{3}}$. The second correct solution, in the negative second quadrant, was only rarely found.

Answer: **(ii)** 10.9°, –169.1°.

Question 5

Almost everyone correctly integrated $\frac{1}{x}$, but few could obtain a multiple of ln(2x + 1) on integrating the second term. Often a promising solution was ended by use of a false rule of the form (ln a – ln b) = $\frac{\ln a}{\ln b}$, or by using approximate values for ln 2, ln 3 and ln 5.

Question 6

Alarmingly, most candidates did not realize that the derivative of a product, y = f(x).g(x) takes the form (fg′+ gf′). Thus y′ was given as a single term, usually as $-\frac{xe^{-x}}{2}$. Those obtaining a correct first derivative often struggled to obtain a correct form for y″. A common error was to solve the equation y′ = 0. The correct derivatives were $e^{-\frac{1}{2}x}(1-\frac{1}{2}x)$ and $e^{-\frac{1}{2}x}\left(\frac{x}{4}-1\right)$.

Answer: (4, $4e^{-2}$).

Question 7

(i) Graphs were generally poor. Many candidates drew that of y = (2 – 2x) as a curve, rather than a line, and few had the intersection as lying between x = 0.5 and x = 1.

(ii) Candidates needed to form a function f(x) = ± (2 – 2x – cos x), to evaluate f(0.5) and f(1.0) and to note that differ in sign.

(iii) Many candidates used cos x = 2 – 2x in an attempt to derive the iterative formula instead of letting n tend to infinity in the iterative formula and obtaining $x = 1 - \frac{1}{2}\cos x$.

(iv) Iteration was usually successful, though many solutions were not rounded to 2 decimal places. Those who erroneously took x to be in degrees (rather than radians) obtained iteratives all equal to 0.5000.

Answer: **(iv)** 0.58.

Question 8

(i) A roundabout argument leading nowhere was common in part **(a)**. In part **(b)**, a common serious error was to cancel the term 'sin' in the expression $\frac{1+\sin x}{1-\sin^2 x}$.

(ii) Differentiation was often poor and not based on use of D{(f(x))^n} = n f(x)$^{n-1}$. f′(x).

(iii) Few correct integrations were performed. The results of parts **(i)(b)** and **(ii)** were required. Many candidates wrongly substituted the limits into the integrand.

Answer: **(iii)** √2.

MATHEMATICS

Paper 9709/03
Paper 3

General comments

The standard of work on this paper varied considerably and resulted in a wide spread of marks. Candidates found this to be one of the more challenging papers set on this syllabus. Nevertheless candidates appeared to have sufficient time to attempt all questions and no question seemed unduly difficult. The questions or parts of questions that were done well were **Question 1** (logarithms), **Question 2** (binomial expansion), **Question 4** (parametric differentiation) and **Question 5(i)** (algebra). Those that were done least well were **Question 6(ii)** (trigonometric equation), **Question 8** (differential equation), and **Question 10** (complex numbers).

In general the presentation of work was good but there were still candidates who presented their work in a double column format. This makes marking difficult for Examiners and it would be helpful if Centres could continue to discourage the practice.

Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often acceptable and that the form given is not necessarily the sole 'correct answer'.

Comments on individual questions

Question 1

This was generally well answered. Some candidates incorrectly assumed $\ln(x + 2) = \ln x + \ln 2$, or $\ln(x + 2) = \ln x \times \ln 2$, but most removed logarithms correctly, reaching the equation $x + 2 = e^2 x$, or equivalent. A significant number failed to solve this correctly for x.

Answer: 0.313 .

Question 2

Most candidates wrote down the correct relevant unsimplified terms of the expansion of $\sqrt{(1-2x)}$, though some took the index to be $-\frac{1}{2}$ or –2 instead of $\frac{1}{2}$. Whatever the expansion, most made sure that the product with $(1 + x)$ contained all the necessary terms. Some confined themselves to finding the coefficient of the term in x^2 instead of the complete expansion up to this term.

Answer: $1-\frac{3}{2}x^2$.

Question 3

The initial differentiation was usually done well using the quotient or product rule. In some cases the quotient rule was misapplied, e.g. by taking $u = \cos x$ instead of $u = e^x$, The presence of e^x in the equation $e^x \cos x + e^x \sin x = 0$ or $e^x \sec x + e^x \sec x \tan x = 0$ unsettled some candidates but the main source of error was the inability to find the root of $\tan x = -1$ in the interval $(-\frac{1}{2}\pi, \frac{1}{2}\pi)$.

Answer: $-\frac{1}{4}\pi$ or –0.785 radians.

Question 4

This was generally very well answered. Candidates who used the double angle formulae after differentiating tended to be more successful than those who substituted at the beginning.

Question 5

There were many correct answers to part **(i)**. In part **(ii)** the correct final answer appeared regularly, but was only very rarely justified properly. Some candidates showed and stated that $2x^2 - 3x + 3$ had no real zeroes but almost always they omitted to either show that it was positive for all x or alternatively examine the sign of $p(x)$ when x took a value other than the sole critical value $-\frac{1}{2}$.

Answers: **(i)** 3; **(ii)** $x < -\frac{1}{2}$.

Question 6

Part **(i)** was usually done well. In part **(ii)** candidates were expected to convert the given equation to the form $13\sin(2\theta + \alpha) = 11$. However common errors were to treat the argument as $2(\theta + \alpha)$, $\theta + \alpha$, or $\theta + 2\alpha$. Some worked with $x + \alpha$ as argument but did not complete the solution by using $x = 2\theta$. Those with a sound approach usually found a root in the interval $0° < \theta < 90°$ but less frequently found the second one in the interval $90° < \theta < 180°$. Moreover, spurious roots quite commonly appeared in the solution.

Answers: **(i)** $13\sin(x + 67.38°)$; **(ii)** 27.4°, 175.2°.

Question 7

(i) This was generally done well. The normal vector $2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$ was sometimes miscopied as $2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ but the correct processes seemed to be well known. Errors in nominating the final acute angle were quite common, with 122.3° or even 57.7° being followed by 32.3° as final answer.

(ii) A minority clearly did not know how to tackle this type of problem. However the majority attacked it with a variety of appropriate methods. Careful checking might have saved the loss of marks caused by algebraic and numerical slips.

Answers: **(i)** 57.7° or 1.01 radians; **(ii)** $\mathbf{r} = 2\mathbf{i} - \mathbf{k} + \lambda(4\mathbf{i} - 7\mathbf{j} + 5\mathbf{k})$.

Question 8

This question was poorly answered. In part **(i)** many candidates, having obtained $\frac{dV}{dh} = 4h^2$, failed to interpret the question as implying that $\frac{dV}{dt} = 20 - kh^2$. Often $\frac{dh}{dt}$ was taken to represent the rate of flow of liquid. Candidates who merely verified that the boundary conditions satisfied the given equation scored zero.

The work in part **(ii)** was also poor. Many candidates seemed to think that verifying the identity for one value of h was enough. There were many incorrect attempts at separating variables in part **(iii)**. The minority who separated correctly often failed to use the identity given in part **(ii)** and made errors in converting $\frac{20h^2}{100 - h^2}$ to an integrable form.

Answer: **(iii)** $t = 100\ln\left(\frac{10+h}{10-h}\right) - 20h$.

Question 9

(i) Most tried to integrate by parts but often made errors in the integration of $e^{\frac{1}{2}x}$. Those who worked correctly seemed to find the final steps from $a^{\frac{1}{2}a}(2a-4)=2$ to the given answer difficult or in some cases impossible.

(ii) Almost all candidates attempted to make sketches of the graph of an exponential function and the appropriate corresponding straight line. Some of the former had the wrong curvature, and some only showed the part of the curve for $x > 0$. Even when both graphs were correctly sketched and provided sufficient evidence for a conclusion to be drawn, many failed to state or indicate that the existence of just one point of intersection implied that the equation had only one root.

(iii) This was satisfactorily answered. There were some candidates who seemed to believe that a statement involving 'positive' and 'negative' was sufficient. However the majority made clear the function they were considering and calculated values as requested, before stating what the change in sign meant.

(iv) Since no iterative formula was given, this part began by testing whether candidates could produce one. Those who stated, or were clearly working with, the formula $x_{n+1}=2+e^{-\frac{1}{2}x_n}$ answered the question well. However some of the candidates who did not state a formula produced sequences that seemed to Examiners to be unrelated to the equation in part **(i)**.

Answer: **(iv)** 2.31 .

Question 10

Parts **(i)** and **(ii)** were generally well answered. Part **(iii)** was hardly ever correctly answered. There seemed a widespread belief that the moduli of the three complex numbers were the lengths of the sides of the triangle whose vertices represented them. Candidates rarely took the results of part **(ii)** to mean that the three vertices were equidistant from the origin and that two pairs subtended $\frac{2}{3}\pi$ at the origin. In part **(iv)** hardly any saw that (4 + 2i)w and $\frac{4+2i}{w}$ were the required numbers. Irrelevant work finding the square roots of 4 + 2i was often seen instead.

Answers: **(i)** 1, $\frac{2}{3}\pi$ (or 2.09 radians); **(ii)** R, $\theta+\frac{2}{3}\pi$; R, $\theta-\frac{2}{3}\pi$;
(iv) $-(2+\sqrt{3})+(2\sqrt{3}-1)i$, $-(2-\sqrt{3})-(2\sqrt{3}+1)i$.

MATHEMATICS

Paper 9709/04
Paper 4

General comments

The work of candidates was generally good and well presented. High marks were scored by many candidates in **Questions 1**, **2**, **3**, **5** and **6**.

Notwithstanding the good work undertaken by candidates, it was disappointing to see that some elementary errors were made with considerable frequency. These included: giving answers for the components of one of the forces instead of those for the resultant of the two forces in **Question 1(ii)**; using the answer in **Question 3(i)** in part **(ii)**, where it has no relevance; calculating the distance *OA* from 0.5×7 or from $\frac{1}{2}g7^2$ in **Question 4**; assuming the time for stage 1 = the time for stage 3 = 200 s in **Question 6(ii)**; obtaining $\frac{0.3t^3}{3}$ from $\int \frac{0.3t^2}{2}\,dt$ in **Question 7(ii)**.

Comments on specific questions

Question 1

(i) This was poorly attempted with many candidates misunderstanding what was required. The most common wrong answer in part **(a)** was $-8\cos\theta$ and this was usually accompanied by the correct answer $8\sin\theta$ in part **(b)**. These are answers for the components of the force of magnitude 8, and the correct answer in part **(b)** is almost certainly fortuitous in most such cases.

(ii) This part of the question was well attempted. Most candidates used $R^2 = X^2 + Y^2$ and solved the resulting equation for $\cos\theta$. Almost all such candidates using this approach scored all three marks, including many who failed to score both marks in part **(i)**. A few candidates solved the simultaneous equations $X = 8\cos\phi$, $Y = 8\sin\phi$ for $\cos\theta$, where ϕ is the angle the resultant of the two given forces makes with the '*x*-axis'. However a much greater number of candidates assumed implicitly from the outset that this angle is θ, not distinguishing it from the θ of the question. Such candidates used $8\cos\theta = \frac{X}{R}$ or $\tan\theta = \frac{Y}{X}$, both of which lead fortuitously to the correct answer, without justifying the use of θ for ϕ. Another very common approach was to use a triangle of forces method. This included either the use of the cosine rule or, less commonly, recognising the triangle as being isosceles and hence $\cos\theta = \frac{1}{2}\times\frac{10}{8}$. Finally a formula method involving $c^2 = a^2 + b^2 + 2ab\cos C$ was frequently used, where *c* is the magnitude of the resultant of two forces of magnitudes *a* and *b*, and the angle between their directions is *C*.

Answers: **(i)(a)** $10 - 8\cos\theta$, **(b)** $8\sin\theta$.

Question 2

This question was well attempted with many candidates scoring high marks. Errors made by candidates included taking the normal reaction force as 20*g*, making sign errors, and omitting the weight component in part **(i)**. Although it was expected that 'a force acts on the block parallel to a line of greatest slope of the plane' would be well understood, a significant number of candidates reacted incorrectly to the words 'up' and 'down'. Such candidates included the normal reaction force on resolving forces in part **(i)**, and the weight instead of its component in part **(ii)**.

Answers: **(i)** 97.8 N; **(ii)** 28.3 N.

Question 3

This was the best attempted of the early questions and a large proportion of candidates scored all four marks in part **(i)**. The given answer in part **(ii)** was achieved by almost all of the candidates, but the level of understanding of why the given lower bound arises from $\frac{18000}{900}$ ranged from non-existent to excellent.

Answer: **(i)** 0.15 ms^{-2}.

Question 4

This question was poorly attempted. Very many candidates failed to find the correct value for the distance *OA*, often inappropriately applying the principle of conservation of energy to obtain 0.0125 metres. Many candidates used 'work done = force $\times$ distance', despite the instruction to use energy. Those who did use energy included many who incorrectly used work done = PE, or work done = KE, or work done = PE – KE.

Answer: 2820 J.

Question 5

This was the best attempted question in the paper. Many candidates started by writing down equations obtained by applying Newton's second law to *A* and to *B*. This leads to a pair of simultaneous equations in three unknowns, *a*, *T* and *m*. Most such candidates were able to find the route to *a* which is independent of *T* and *m* and, after finding the value of *a*, were able to find *T* and *m*.

Answers: **(i)(a)** 2.5 ms^{-2}, **(b)** 3.75 N; **(ii)** 0.3.

Question 6

Almost all candidates scored both of the available marks in part **(i)**, but part **(ii)** was poorly attempted. Frequently occurring wrong answers included 20 ms^{-1} (from $\frac{20000}{1000}$), 30 ms^{-1} (from 0 + 0.15 $\times$ 200), 40 ms^{-1} (from $\frac{20000}{1000} = \frac{0+v}{2}$), and 25 ms^{-1} using the assumption that the times for the first and third stages are equal.

In part **(iii)** many candidates used correct methods but some assumed that the time for stage 1 = the time for stage 3 = 200 s.

Answers: **(ii)** 25; **(iii)** 2920 m.

Question 7

Many candidates scored full marks in **part (i)**. Most candidates realised the need to integrate in part **(ii)** and many candidates did so twice to obtain an expression for the distance $x(t)$. Integration was usually executed accurately. However some candidates integrated only once to obtain what they declared to be an expression for $x(t)$.

Few candidates dealt with the constants of integration or with the evaluation at the final stage correctly. The first constant of integration was more frequently found to be 0.0375 (from $5 - 10 \times 0.5 + 0.15 \times 0.5^2$), or zero, than the correct value of 5. Similarly the second was frequently found to be a value based on $x(0.5) = 0$ instead of the correct value of 0 or 1.25 (depending on whether the $x(t)$ represents the displacement from *A* or from *O*).

At the final stage more candidates substituted $t = 3$ than substituted the appropriate value of $t = 2.5$. Almost all candidates who evaluated a definite integral at the second stage found $\int_{0.5}^{3} v(t)\,\mathrm{d}t$ instead of $\int_{0}^{2.5} v(t)\,\mathrm{d}t$.

Answers: **(i)** 5 ms^{-1}, 0.5 s; **(ii)** 44.2 m.

MATHEMATICS

Paper 9709/05

Paper 5

General comments

The more able candidates were able to score well, but unfortunately some candidates scored low marks and were clearly not ready for the examination at this level.

The work from some candidates was poorly presented and often difficult to read. However more candidates are drawing clear diagrams to help them with their solutions.

Only a few candidates used premature approximation and rounded to less than 3 significant figures. The question paper clearly states that $g = 10$ should be used and candidates rarely used $g = 9.8$ or 9.81.

The more able candidates scored well on **Questions 1** to **4**. **Questions 5**, **6** and **7** proved to be more of a challenge with **Question 5** found to be the most difficult question on the paper, requiring candidates to use trigonometry to work out various distances.

Some candidates had a limited understanding of how to take moments.

Comments on specific questions

Question 1

Some candidates tried to use an energy equation which was not productive. At times $x = 6$ was used for the extension in $T = \lambda \frac{x}{l}$. Sometimes $T = 8a$ only was seen instead of $T - 8g = 8a$.

Answer: 4 ms^{-2}.

Question 2

(i) Unfortunately the centre of gravity of the cone from the base was sometimes taken as $\frac{h}{3}$ or $\frac{3h}{4}$.

(ii) Most candidates obtained $\alpha = 39.8$, but quite a number found the complement, 50.2.

Answers: **(i)** 48; **(ii)** 39.8.

Question 3

(i) Most of the candidates gained full credit.

(ii) Often the integration was correctly done. On occasions either v or v^2 was seen instead of $\frac{v^2}{2}$. Some candidates did not know how to integrate $16e^{-x}$.

Answer: **(ii)** 5.33 ms^{-1}.

Question 4

(i) Some candidates made part **(b)** rather long winded by finding ω first and then using $v = r\omega$.

(ii) Again some candidates made this part long winded by first finding v and then using $\omega = \frac{v}{r}$. Sometimes candidates used $2.5 - 0.671 = 0.15 \times 0.2\omega^2$ or $2.5 = 0.25 \times 0.2\omega^2$.

Answers: **(i)(a)** 1.5 N, **(b)** 0.671 N; **(ii)** 9.13 rad s^{-1}.

Question 5

This was found to be the most difficult question on the paper.

(i) This part needed a really good clear diagram to help the candidate find the distances and the forces needed to solve the problem. Most candidates attempted to take moments about *D*. Often $0.8T = 350 \times 0.6\cos 20°$ was seen instead of $0.8T = 360 \times 0.6\cos 20° - 350 \times 0.4\sin 20°$. Some candidates attempted this part by simply trying to resolve in numerous directions. This approach was not productive.

(ii) Candidates made a better attempt at this part of the question. Candidates tried to find *F* and *R* and then used $F = \mu R$. Unfortunately *F* and *R* were often incorrectly calculated.

Answer: **(ii)** 0.424.

Question 6

(i) Most candidates attempted to use an energy equation. Many candidates often found $v^2 = 40 + 10x - 5x^2$ but then simply stated that $v^2 = 45 - 5(x - 1)^2$. This was too big a step to take since the answer is given on the question paper.

(ii) Too many candidates found $x = 4$ but then omitted to add 4 to give the distance at the lowest point.

(iii) This was generally well done. Some candidates substituted $x = 0$ instead of $x = 1$ into $v^2 = 45 - 5(x - 1)^2$.

Answers: **(ii)** 8 m; **(iii)** 6.71 ms^{-1}.

Question 7

(i) Quite a number of candidates did not express *x* and *y* in terms of *V* and *t*. These candidates simply quoted the equation of the trajectory and substituted $\theta = 60°$ into it.

(ii) This part of the question was often done correctly. Some candidates had difficulty in manipulating the expression to make *V* the subject.

(iii) Only a handful of candidates differentiated the answer in part **(i)** to find $\frac{dy}{dx} = \sqrt{3} - \frac{40x}{V^2}$ and then went on to find the required direction of motion of *P* at the instant it passes through *A*.

Answers: **(i)** $x = Vt\cos 60°$, $y = \sqrt{3}x - \frac{20x^2}{V_2}$; **(ii)** 29.7; **(iii)** 55.3° downward from the horizontal.

MATHEMATICS

Paper 9709/06

Paper 6

General comments

The paper proved accessible to most candidates. There was no problem with shortage of time, and almost everybody attempted questions in numerical order. It was pleasing to see that the majority of candidates worked with an appropriate number of significant figures and therefore did not lose any accuracy marks. However, those who worked with 3 significant figures, i.e. with premature rounding, invariably lost the final accuracy mark because their answer was not accurate to 3 significant figures.

Comments on specific questions

Question 1

This was usually answered well. Most errors arose from confusion over Σx^2 and $(\Sigma x)^2$. This question was one example where premature rounding, which gave an answer of 4.56 instead of 4.57, was evident.

Answers: 38.4 mm, 4.57 mm.

Question 2

There were many good solutions obtaining full marks. Some candidates tried to use the binomial distribution which gained no marks as it is not an approximation, which the question asked for. There were the usual number of errors concerning standard deviation and variance, and continuity corrections. It is pleasing to see candidates are using diagrams more often to determine the tail.

Answer: 0.652.

Question 3

Generally, candidates who managed **Question 2** successfully also coped well with this normal distribution question. Candidates who used a continuity correction here did not gain the method mark. There was also some confusion over $1-\Phi(z)$ and $\Phi(1-z)$. Candidates who used 1 – 1.22 did not gain credit for method. Some candidates found the negative numbers difficult to manage, and had –15.1 to the right of 0 on the normal curve in part **(i)**, whilst in part **(ii)** 0° C was to the right of the mean. Incorrect answers of 0.0277, 0.0278, etc. were seen due to premature rounding and again these answers lost the final mark as they are not accurate to 3 significant figures.

Answers: **(i)** 0.0276; **(ii)** 7.72.

Question 4

This question was poorly answered. Most candidates gave an answer of 12! to the first part, failing to appreciate that some of the houses were the same. For part **(ii)** many candidates could not determine the number in each group, and if they did, they invariably added instead of multiplied for the final probability. In part **(iii)** many errors arose from candidates using ${}^{10}C_2$ instead of ${}^{7}C_2$.

Answers: **(i)** 831 600; **(ii)** 900; **(iii)** 126.

Question 5

This question was generally well answered. There were some instances of wrong stems of 1, 2, 3, etc. or 100, 110, 120, etc. and some leaves were not placed in columns but were spaced out. In part **(ii)** a few candidates did not add 1 to the number of items (15) and thus found the quartiles wrongly. These answers could have been read off and written down in 3 lines but many candidates took over one side of working to arrive at the answers. Almost everybody knew what a box-and-whisker plot was, but nearly everybody lost credit because 'pulse rate' or 'beats per minute' was not seen on the graph. It is essential to put the units in when using statistical illustrations. A few drew whisker lines right through the box. Boxes should be empty, in the order of 1 cm high (not 5 or 6 as was often seen) and the ends of whiskers should be shown by a mark of some sort. A ruler should be used.

Answers: **(ii)** 125, 115, 145.

Question 6

Many candidates scored no marks for this question. A three-stage tree diagram proved problematic for many. Some solutions had an incorrect part **(i)** but then recovered in parts **(ii)** and **(iii)**, whilst others had a correct part **(i)** but only had two probabilities in later parts. Overall there was very little correlation in this question between which of parts **(i), (ii)** or **(iii)** candidates got correct. Part **(iv)** however was universally poorly done with most candidates only finding one option for the numerator instead of two, and those who did find two options and summed them correctly then failed to divide by their answer to part **(iii)**. Able candidates scored full marks and this question served as a good discriminator.

Answers: **(ii)** 0.224; **(iii)** 0.392; **(iv)** 0.633.

Question 7

Many good candidates answered this question well. The weaker candidates failed either to find the probability of throwing an odd number or to recognise a binomial situation, or both. Having negotiated both those hurdles, errors then occurred in finding the probability of at least 7 odd numbers. Candidates found P(7), P(8), 1 – any combination of the two, and so on. Parts **(ii)** and **(iii)** were straightforward and almost everybody picked up some marks here. Part **(iv)** needed a little more understanding which was lacking in many of the weaker candidates.

Answers: **(i)** 0.195; **(ii)** 2, $\frac{1}{36}$; 4, $\frac{2}{36}$; 6, $\frac{5}{36}$; 7, $\frac{4}{36}$; 8, $\frac{4}{36}$; 9, $\frac{4}{36}$; 10, $\frac{4}{36}$; 11, $\frac{8}{36}$, 12, $\frac{4}{36}$;
(iii) $\frac{26}{3}$; **(iv)** $\frac{5}{9}$.

MATHEMATICS

Paper 9709/07
Paper 7

General comments

This paper proved to be reasonably straightforward, with candidates able to demonstrate and apply their knowledge in the situations presented. There were many good scripts, with few candidates appearing totally unprepared for the paper. In general, candidates scored well on **Questions 7** and **4(i)**, **(ii)**, whilst **Questions 1**, **2** and **4(iii)** proved more demanding. It was pleasing to note that **Question 5**, which required knowledge of Type I and Type II errors, was reasonably well attempted by some candidates. In previous papers, this has not always been the case.

Accuracy, as always, caused loss of marks for some candidates; there were a few cases of candidates not adhering to the accuracy required, either by rounding too early in the question or by giving a final answer to only 2, or even 1, significant figures. This was particularly noticeable in **Question 5**. Lack of time did not seem to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also many very good and complete answers.

Comments on specific questions

Question 1

This question was only well attempted by a minority of candidates. Some candidates gave wholly incorrect and irrelevant explanations, such as considering the musical talent of the children. In part **(i)** some candidates realised that the method was unsatisfactory, but did not give a full enough answer to obtain all available marks. A statement such as 'not all children have the same chance of being chosen', whilst a true statement, does not explain why they do not have equal chance and is not related to the question in any way. Similarly in part **(ii)** many candidates did not give a detailed enough answer to explain a full and valid method.

Question 2

Weaker candidates did not score well on this question. Common errors included using a one-tail test, using 15.7 rather than $\frac{15.7}{\sqrt{3}}$ when finding the test statistic. Some candidates failed to find the average time of the given sample of size 3 and attempted to find 3 separate test statistics, showing a lack of understanding of what was required. Other errors included incorrect comparisons (e.g. incorrect z critical, or even a comparison of an area with a z-value) and in some cases no comparison was shown to justify a final conclusion. Any final statement in a hypothesis test such as this must be justified by a relevant comparison. Many candidates wrote their null and alternative hypotheses as $H_0 = 42$ and $H_1 \neq 42$, omitting μ. It was disappointing to find that candidates, who sometimes did the rest of the question successfully, lost marks in this way.

Answer: Teacher's estimate can be accepted.

Question 3

Many candidates were able to offer a good solution to this question, and a good attempt was often made even by weaker candidates. The most common error, as is always the case on this type of question, was an incorrect variance obtained from $2^2 \times 1.6^2 + 10^2 \times 0.4^2$ rather than the correct variance of $2 \times 1.6^2 + 10 \times 0.4^2$

Answer: 0.350.

Question 4

As mentioned above, candidates found parts **(i)** and **(ii)** of this question straightforward. There were few consistent errors other than the usual confusion between biased and unbiased estimates, and the two different formulas for the unbiased estimate of the variance. The most successful candidates used the formula given in the formula list. Many candidates found the correct confidence interval; the most common errors noted were incorrect *z*-values or an incorrect formula.

Part **(iii)**, however, was poorly attempted and omitted completely by many candidates. It was surprising to note, that whilst candidates could find a confidence interval, they were unable to interpret what it meant.

Answers: **(i)** 4.27, 0.00793; **(ii)** (4.25, 4.29); **(iii)** 9.

Question 5

It was pleasing to note that some candidates correctly identified the probability of a Type I and a Type II error in the given situation. There were still cases where candidates quoted what was meant by these errors, and were unable to go any further and relate this knowledge to the question, but only weaker candidates did not actually make any attempt at finding probabilities. In parts **(i)** and **(ii)** many candidates reversed the probabilities finding P(0) instead of P(1 or more) in part **(i)** and P(1 or more) instead of P(0) in part **(ii)**. Many candidates correctly used a Poisson approximation in part **(iii)**, though the correct value of λ (0.288) was not always found.

Answers: **(i)** 0.0202; **(ii)** 0.972; **(iii)** 0.0311.

Question 6

Most candidates correctly found the answer to part **(i)**, though some incorrectly felt the answer required rounding to the nearest whole number. Parts **(ii)** and **(iii)** were not always well answered. Some candidates did not realise the need to multiply two Poisson probabilities in **part (ii)**. In part **(iii)**, whilst many candidates calculated 1 – P(0), some did not give this as a final answer, and some did not use the correct value of λ. Similarly in part **(iv)** candidates did not always calculate a new value of λ and merely used the value of 1.8.

Answers: **(i)** 1.15; **(ii)** 0.216; **(iii)** 0.784; **(iv)** 0.776.

Question 7

This was a well attempted question, even by weaker candidates. Most candidates correctly showed that *k* was $\frac{3}{14}$, and there were few cases of candidates showing insufficient working. On the whole, attempts at integration were good, though some candidates used unnecessarily long methods (integration by parts). There was an occasional confusion between part **(ii)** requiring the mean and part **(iii)** requiring the median. On the whole errors were mainly calculation errors, though an error of limits in part **(iii)** was quite common with candidates integrating from '0 to *m*' rather that from '1 to *m*'. Many candidates used a correct method for part **(iv)**; once again errors were mainly from incorrect calculations rather than from errors in method.

Answers: **(ii)** 2.66 hours; **(iii)** 2.73 hours; **(iv)** 0.0243.