CONTENTS

FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned**.

MATHEMATICS

GCE Advanced Level and GCE Advanced Subsidiary Level

Paper 9709/01
Paper 1

General comments

Most candidates found this paper to be well within their grasp and there were many excellent scripts. The standards of algebra and numeracy were good and most scripts were well presented and easy to mark. It was particularly pleasing to see most candidates showing full working as this is an advantage for both candidates and Examiners.

Comments on specific questions

Question 1

This proved to be a good starting question, presenting little difficulty to the majority of candidates. Apart from the occasional algebraic or arithmetical error, the solution of the quadratic, in either *x* or in *y*, was usually completely correct.

Answers: (1.5, 8) and (4, 3).

Question 2

Part **(i)** was usually correctly answered following use of the identity '$\sin^2\theta + \cos^2\theta = 1$', though a few candidates ignored the first request and never obtained a quadratic in *x*. A few recognised the equation as a quadratic in $\sin^2\theta$, but following the solution of the quadratic, expressed the roots as $\sin\theta$. However, by far the greatest error was to deduce that the solution of $\sin^2\theta = 0.25$ was $\sin\theta = 0.5$ rather than ±0.5, thereby omitting the solutions $\theta = 210°$ and 330°.

Answers: **(ii)** 30°, 150°, 210°, 330°.

Question 3

(a) Only about a half of all attempts realised that \$3726 was the sum of all the payments and not the final payment and use of $u_n = 3276$ was widespread. Of those using $S_n = 3726$, the vast majority correctly substituted $a = 60$ and $n = 48$ to deduce that $d = \frac{3}{4}$.

(b) This was very well answered with virtually all candidates correctly using the formula for the sum to infinity of a geometric progression. Evaluating *r* from *a* and *n* presented few problems, though occasional $r = 1.5$ was obtained rather than $r = \frac{2}{3}$. It was obvious from such attempts that candidates also failed to realise the condition '$|r| < 1$'.

Answers: **(a)** \$61.50; **(b)** 18.

Question 4

(i) Most candidates realised the need to integrate and the standard of integration was good. A significant number however failed to appreciate the need to use the given point (1, 5) to evaluate the constant of integration. It was still common to see weaker candidates taking *m* as $\frac{dy}{dx}$ and substituting into $y = mx + c$.

(ii) This caused a few problems with many candidates failing to appreciate the need to solve the inequality, $\frac{dy}{dx} > 0$. The solution of the quadratic was accurately carried out and the solution of the inequality was better than in previous years. There were many solutions, however, in which the solution of $(x - 1)(3x - 1) > 0$ was given as '$x > 1 > \frac{1}{3}$'.

Answers: **(i)** $y = x^3 - 2x^2 + x + 5$; **(ii)** $x < \frac{1}{3}$ and $x > 1$.

Question 5

This was very well answered and the majority of candidates obtained full marks. The most common error was to assume that *AB* and *BC* were perpendicular leading to a gradient of $-\frac{1}{3}$ for *BC*. Most of these candidates usually continued by assuming that the gradient of *CD* was 3. Apart from this, the standard of algebra required to find the equations of lines and then to solve the simultaneous equations was very good.

Answers: **(i)** $2y = x + 8$, $y + 2x = 29$; **(ii)** (10, 9).

Question 6

Parts **(i)** and **(ii)** were well answered, but part **(iii)** presented candidates with more serious problems. Candidates were generally correct in linking perimeter with arc length to evaluate the given answer for θ, though there were several attempts in which the difference between arc length and perimeter was not fully appreciated. Most candidates then proceeded to obtain a correct expression for the area of the sector in terms of *r*. In part **(iii)** however, only a minority of candidates realised the implication of the word 'chord' and realised the need to calculate the straight distance *PQ* rather than the arc *PQ*. Of those correctly attempting part **(ii)**, attempts were split between those using the cosine rule and those splitting the isosceles triangle into two 90° triangles.

Answers: **(ii)** $A = 10r - r^2$; **(iii)** 3.96 cm.

Question 7

Solutions to this type of problem have improved considerably over the past few papers, but there were still a significant number of solutions in which the dimensions of the prism were ignored in finding expressions for $\overrightarrow{MC}$ and $\overrightarrow{MN}$ in part **(ii)**. Using $\overrightarrow{CM}$ for $\overrightarrow{MC}$ remains a common error but there were only a few solutions in which $\overrightarrow{MC}$ was taken as $\overrightarrow{OM} + \overrightarrow{OC}$. The use of techniques used in part **(iii)** was excellent, but it was surprising that many candidates deliberately ignored the minus sign or obtained an obtuse angle and then gave the answer as an acute angle.

Answers: **(i)** 4 units; **(ii)** $\overrightarrow{MC}$ = 3**i** − 6**j** − 4**k**, $\overrightarrow{MN}$ = 6**j** − 4**k**; **(iii)** −20, 111°.

Question 8

Most candidates realised that $2x^2y = 72$ and that $A = 4x^2 + 6xy$, but many failed to realise the need to substitute for *y* in order to obtain the given expression for *A* in terms of *x.* The differentiation of *A* and solution of $\frac{dA}{dx} = 0$ was accurate and most candidates were confident in determining the nature of the stationary point, usually by finding the sign of the second differential. Candidates need to read questions carefully since over a third of all attempts failed to give the stationary value of *A* as requested at the beginning of part **(iii)**.

Answers: **(i)** $y = \frac{36}{x^2}$; **(ii)** $x = 3$; **(iii)** A = 108 cm^2, minimum.

Question 9

(i) The differentiation of $\frac{8}{3x+2}$ was generally accurate, though many failed to include the differential of $(3x+2)$. Several candidates attempted to use the quotient rule and often failed to obtain a correct answer through assuming that $\frac{d}{dx}(8) = 1$. A surprising number also quoted the incorrect quotient formula. Having obtained a numerical value for $\frac{dy}{dx}$, most correctly found the equation of the tangent. Fewer candidates than usual expressed '*m*' algebraically as $\frac{dy}{dx}$. Most also realised the need to set *y* to 0, prior to finding the length *DC* and finally the area of the triangle.

(ii) Most candidates realised the need to use the correct formula for the volume of rotation of a curve, but the standard of integration was poor. Only about a half of all attempts realised that $\int (3x+2)^{-2} dx = \frac{(3x+2)^{-1}}{-3}$ (+*c*). Many failed to realise the need to include '÷3' and several others finished with an incorrect power of $(3x+2)$. Surprisingly, a large number were seen in which other functions of *x* appeared in the answer. Use of limits was generally correct, though about a quarter of all attempts automatically assumed that the value at the lower limit of 0 could be ignored.

Answers: **(i)** $8y + 3x = 14$.

Question 10

Overall the attempts at this question were very pleasing, with many completely correct solutions. Part **(i)** presented few problems and it was very rare to see gf being used instead of fg. The solution of $\frac{8}{2-x} - 5 = 7$ was usually correct, but such errors as multiplying through by $(2-x)$ to obtain $8 - 5 = 7(2-x)$ were seen. Part **(ii)** produced excellent answers with virtually all candidates correctly obtaining an expression for f^{-1} and coping comfortably with g^{-1} apart from a few errors in sign. In part **(iii)** most candidates equated f^{-1} with g^{-1} to obtain a quadratic equation in *x*. About a half of all candidates then attempted to use the discriminant '$b^2 - 4ac$', and most realised that a negative answer implied no real roots. Of those attempting to solve the equation by the quadratic formula, most stopped at an expression containing $\sqrt{-31}$ without explaining why such an expression was non-real. Examiners cannot assume such facts without explanation. The graphs in part **(iv)** were generally well done, though some candidates spent considerable (and unnecessary) time on accurate graphs. A surprising number of candidates failed to recognise that the graphs of both $y = 2x - 5$ and $2y = x + 5$ are straight lines, though at least a half of all attempts recognised the symmetry about the line *y* = *x*.

Answers: **(i)** $1\frac{1}{3}$; **(ii)** $f^{-1}(x) = \frac{1}{2}(x+5)$, $g^{-1}(x) = \frac{2x-4}{x}$; **(iv)** Sketch - symmetry about *y* = *x*.

Paper 9709/02

Paper 2

General comments

The Examiners were disappointed by the overall standard of the scripts. Previous Reports have stressed the importance of candidates familiarising themselves with the list of formulae MF9 (especially those sections dealing with the formulae and results for differentiation and integration), and of working carefully through past 9709/02 Papers. There was little evidence that this advice had been followed by the majority of candidates.

Where questions were structured so that the result(s) of early part(s) were crucial to attempting successfully later parts of the problem - especially so for **Questions 4**, **6 (iii)** and **7** - candidates generally failed to note the connection. Questions poorly attempted included **Questions 2**, **5 (i)**, **6 (iii)** and **7 (ii)**, **(iii)** and **(iv)**. Candidates were at ease, on the other hand, with **Questions 3 (i)**, **4 (i)**, **6 (i)** and **7 (i)**. **Questions 6** and **7**, where a substantial number of marks were available, produced very low scores overall.

Work was neat and well presented and the Examiners were impressed by the clarity of candidates' reasoning. There was no evidence of the time available being inadequate, and where questions were not fully attempted, this appeared due to a lack of confidence. Preparation for this paper remains somewhat lacking and the Examiners stress the need for addressing all, and not just parts, of the syllabus.

Comments on specific questions

Question 1

A majority of candidates scored the first two marks, but then adopted inequality signs, or one sign, the reverse of those required. A shrewd technique adopted by many was to note that $x = \frac{8}{3}$ satisfies the inequality, and hence this value must be included in the solution. Most candidates squared each side, but many, forgot that $|a| < b$ yields $a^2 < b^2$ and not $a^2 < b$. Those who adopted the $-2 < 8 - 3x < 2$ approach usually failed to get beyond a single mark for noting that x = 2 was a critical value.

Answer: $2 < x < \frac{10}{3}$.

Question 2

As on previous occasions when this type of question has been set, most candidates failed to notice that the vertical axis represents values of ln *y*, not *y*. The key feature to note is that ln *y* = ln *k* – *x*ln *a* and hence there is a linear relationship between ln *y* and *x*, with ln *k* being the intercept on the vertical axis (i.e. the value of ln *k* when $x = 0$) and –ln *a* being the gradient of the line, calculated as -0.75 by considering the graph. A less popular, alternative treatment consisted of calculating the values of *y* corresponding to the key values of ln *y* (namely 2, 1.4, 1.1 and 0.5) and feeding two values of *y* and the corresponding *x*-values into the formula $y = k(a^{-x})$; this was generally attempted successfully by those few who preferred this method.

Answers: *a* = 2.12; *k* = 9.97 .

Question 3

(i) This presented few problems except for those who set $x = +1$ in the quartic expression, or those unable to solve the equation $1 - 6 - 1 + a = 0$.

(ii) Few candidates failed to correctly check that $(x - 2)$ was a factor of $f(x)$. However, many then failed to obtain a second correct cubic factor, via long division or by inspection. Others noted that $(x + 1)(x - 2) = x^2 - x - 2$ was a factor of $f(x)$, but struggled to correctly ascertain the other quadratic factor. All errors essentially were due to poor arithmetic.

Answers **(i)** 6; **(ii)** $f(x) = (x + 1)(x - 2)(x^2 - x - 3)$.

Question 4

(i) Although many candidates correctly noted that $R\cos\alpha = 1$, $R\sin\alpha = \sqrt{3}$, a surprising number of solutions featured a wrong value for R and/or a failure to solve the equation $\tan\alpha = \sqrt{3}$. The question asks for α to satisfy $0 < \alpha < \frac{1}{2}\pi$, and hence α is in **radians**; at least half of solutions gave α as 60°, rather than $\frac{\pi}{3}$ radians.

(ii) Many candidates failed to use their result from part **(i)** to note that $R\cos(\theta - \alpha)$ = $\sqrt{2}$ so that $(\theta - \alpha) = \cos^{-1}\left(\frac{\sqrt{2}}{R}\right)$, etc. In seeking a second solution, many values in the wrong quadrants were produced; $(\theta - \alpha)$ has 2 values, in the first and fourth quadrants, but many candidates were convinced that any second θ-value must be equal to $\pi - \theta_1$ or $\pi + \theta_1$ where θ_1 is the solution in the first quadrant.

Answers: **(i)** $R = 2$, α = $\frac{1}{3}\pi$; **(ii)** $\frac{1}{12}\pi$.

Question 5

(i) Very few correct pairs of graphs were seen. Many pairs occupied only the first and third quadrants; the single negative root lies in the second quadrant.

(ii) The technique required here is to define $\mathrm{f}(x)$ as equal to $\pm(x^2 - 2^x)$ and to note that $\mathrm{f}(-1.0)$ and $\mathrm{f}(-0.5)$ have different signs, indicating that $\mathrm{f}(x) = 0$ somewhere between $x = 1.0$ and $x = -0.5$. Candidates often simply looked at the values of x^2 and 2^x at $x = -0.5$ and $x = -1.0$ and tried unconvincingly to prove the proposition.

(iii) Candidates were asked to determine a root correct to two significant figures, but this requires working to at least three, and preferably four, significant figures at the stages preceding a final value; few candidates did so. From part **(ii)**, it was given that the root lies between $x = -1.0$ and x = –0.5. However, a significant proportion of candidates started correctly, at –0.5, –0.75 or –1.0 and after only one iteration were straying far from the interval $-1 < x < -0.5$. It was surprising that such solutions were not quickly seen as non-viable, with candidates struggling to calculate $x_2 = \sqrt{2^{x_1}}$, with $x_1 = -0.5, -0.75$ or -1.0.

Answer: **(iii)** $x = -0.77$.

Question 6

(i) No more than half the solutions correctly calculated that A was such that y = 0 and hence $(4 - x) = 0$ there, and that, at B, $x = 0$ and hence $y = (4 - 0)e^0$ there.

(ii) After correctly differentiating $(4 - x)e^{+x}$ to get $\frac{dy}{dx} = e^{+x}\{+(4 - x) - 1\}$, many candidates then set $x = 0$, instead of setting $\frac{dy}{dx} = 0$. Others believed that $e^{+x} = 0$ gave a correct solution for x. The actual differentiation of y was generally good, though some sign errors were seen and a few derivatives featured only one term, using the incorrect form $\frac{d}{dx}$\{f(x).g(x)\} = f ′(x).g ′(x).

(iii) Virtually no-one scored any marks, and few even attempted this part. It is required to note firstly that P has coordinates (p, (4 – p)e^p) and hence the gradient of the line OP is $\left(\frac{4 - p}{p}\right)e^p$. This value could then be equated to that found in part **(ii)** for the gradient, namely $(3 - p)e^p$ at P.

Answers: **(i)** A(4, 0), B(0, 4); **(ii)** 3; **(iii)** 2.

Question 7

(i) There was much excellent differentiation, either of $\frac{\cos x}{\sin x}$ or of $(\tan x)^{-1}$ using the quotient or 'function of a function' rule. However, many candidates, in effect, only quoted the result.

(ii) Few candidates noted that, using the result of part **(i)** in reverse, $\int \operatorname{cosec}^2 x \, dx = \cot x \; (+ c)$. At best, +cot *x* was quoted by most who had the correct basic idea.

(iii) Here $\cot^2 x \equiv \operatorname{cosec}^2 x - 1$ is used and in part **(ii)** gives the result for $\int \operatorname{cosec}^2 x \, dx$, leaving only the integration of –1 to do.

(iv) The denominator reduces to $2\sin^2 x$ and hence the integral to $\frac{1}{2}\int \operatorname{cosec}^2 x \, dx$, again part **(ii)** giving the key to the result.

In parts **(ii)**, **(iii)** and **(iv)** a host of incorrect forms were quoted, none of which corresponded to results in list MF9.

Answers: **(iii)** $\left(\sqrt{3} - \frac{1}{3}\pi\right)$; **(iv)** $\frac{1}{2}\sqrt{3}$.

Paper 9709/03
Paper 3

General comments

There was considerable variation in the standard of work on this paper and a corresponding spread of marks from zero to full marks. The paper appeared to be accessible to candidates who were well prepared and no question seemed to be of undue difficulty, though correct solutions to the final part of **Question 7** (complex numbers) were rare. Adequately prepared candidates seemed to have sufficient time to attempt all questions and presented their work well. However Examiners found that there were some very weak, often untidy, scripts from candidates who clearly lacked the preparation necessary for work at the level demanded by this paper. All questions discriminated to some extent. Overall, the least well answered questions were **Question 4** (implicit differentiation) and **Question 7** (complex numbers). By contrast, **Question 3** (trigonometric equation) was usually answered very well and Examiners were impressed by the work of many candidates on **Question 10** (vector geometry).

The detailed comments that follow inevitably refer to common errors and can lead to a cumulative impression of poor work on a difficult paper. In fact there were many scripts showing a good and sometimes excellent understanding of all the topics being tested.

Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often possible and that the form given is not necessarily the sole 'correct answer'.

Comments on specific questions

Question 1

This was fairly well answered by a variety of methods. Most candidates were able to use logarithms correctly in attempting to find at least one of the critical values.

Answer: $1.58 < x < 3.70$.

Question 2

The most popular method was to remove a numerical factor and expand $\left(1+\frac{1}{2}x^2\right)^{-2}$. The binomial expansion was often correct but the numerical factor was quite frequently wrong and sometimes omitted or lost in the course of the solution. The minority who attempted to expand the given expression directly tended to be less successful.

Answer: $\frac{1}{4}-\frac{1}{4}x^2+\frac{3}{16}x^4$.

Question 3

This was very well answered and solutions were often completely correct. Most errors were associated with the solution of the equation cos $\theta = -1$. Often $\theta = 0°$ was included as a solution, but it was equally popular to assert that the equation has no solutions.

Answers: 33.6°, 180°.

Question 4

In part **(i)**, there were many good attempts at implicit differentiation, the main error being the omission of the minus sign when giving the final answer. Candidates who first rearranged the equation and attempted to remove some of the square roots were often unsuccessful. Failure to square correctly led to worthless solutions based on incorrect relations such as $y = a - x$ or $y = a + x$.

Part **(ii)** was poorly done. Relatively few candidates appeared to understand how to obtain the coordinates of P. Those that did have a valid method often made errors in handling square roots. In forming the equation of the tangent at P, a persistent error was the use of a general gradient rather than the specific gradient at P.

Answers: **(i)** $-\sqrt{\frac{y}{x}}$; **(ii)** $x+y=\frac{1}{2}a$.

Question 5

In part **(i)**, most candidates sketched y = sec x and $y=3-x^2$, but some worked with acceptable alternatives after rearranging the equation. Candidates should be reminded of the importance of labelling sketches and thus making it clear to Examiners what is being attempted. The quality of the sketches was generally poor with, for example, y = sec x rarely fully correct and $y=3-x^2$ commonly presented as a straight line. Examiners remarked that candidates seemed better prepared for part **(ii)** than in previous questions on this topic. Part **(iii)** was frequently correctly done. The most common error here was to carry out the calculations with the calculator in degree mode rather than in radian mode. Here, as in part **(ii)**, there was evidence that some candidates did not have a correct appreciation of the notation $\cos^{-1}x$.

Answer: **(iii)** 1.03.

Question 6

Part **(i)** was generally quite well answered. Most candidates used the product rule correctly and solved the linear equation in x resulting from setting the derivative to zero and removing the non-zero common factor of e^{-2x}. However for some candidates this common factor presented problems and led to them making a variety of algebraic errors. Examiners also noted that a minority seemed to believe that the turning point occurred when the second derivative was zero. Most candidates attempted to apply the method of integration by parts correctly in part **(ii)** and inserted the correct limits x = 0 and x = 3. However many otherwise sound solutions lost marks because a sufficiently diligent check for sign errors was not made throughout the working.

Answers: **(i)** $3\frac{1}{2}$; **(ii)** $\frac{1}{4}(5+e^{-6})$.

Question 7

Part **(i)** was well answered. In part **(ii)**, the point corresponding to *u* was usually plotted accurately, and many candidates demonstrated some knowledge of the correct locus for *z*. However, there were often errors in the sketch. For example, it was common for the circle to have a radius greater than 2, and candidates who had different scales on their axes usually failed to take this fact into account. Very few candidates showed any indication that they had a method for completing part **(iii)**. Credit was given to the small number who at least identified the relevant point by drawing the appropriate tangent to their circle. But of this group of candidates there were only a few who went on to calculate the required argument.

Answers: **(i)** 1 + 2i; **(iii)** 126.9°.

Question 8

Even though the correct form of partial fractions was given, a substantial number of candidates ignored *A*, the first term. A similar error of principle was quite often made by those who chose to divide first. They usually found *A* = 1, and obtained a quadratic remainder, but then set the remaining two partial fractions equal to f(*x*), i.e. they failed to use their remainder as the new numerator. However most candidates were clearly familiar with a method for evaluating constants and there were a pleasing number of fully correct solutions. In part **(ii)**, much of the integration was good. Those who had failed to obtain *D* = 0 usually encountered severe difficulties here and wasted time that might have been better spent looking for the error in part **(i)** that got them into this situation. Examiners remarked that some candidates with correct solutions did not show sufficient evidence of how they obtained the final (given) answer.

Answer: **(i)** $1-\frac{1}{x-1}+\frac{2x}{x^2+1}$.

Question 9

Most candidates separated variables correctly and showed a sound understanding of the methods needed for each part. Many solutions to part **(i)** were correct, apart perhaps from a sign error, and usually included a constant of integration. In this question, as in **Question 4** above, Examiners reported that candidates frequently made errors when manipulating or removing square roots.

Answers: **(i)** $2\sqrt{(P-A)}=-kt+c$; **(iii)** 4; **(iv)** $P=\frac{1}{4}A(4+(4-t)^2)$.

Question 10

This was well answered even by candidates who had not scored particularly well on earlier questions.

There were many successful solutions to part **(i)**. Having used two component equations to calculate *s* or *t*, many candidates went on to calculate the other parameter and check that the third equation was satisfied. However, some omitted this step or else checked in one of the equations already used. Also some forgot to conclude by stating the position vector of the point of intersection.

A variety of methods were seen in part **(ii)**. Though it is not in the syllabus, some candidates used the vector product correctly. The most popular method was to set up two equations in *a*, *b*, *c* and, having obtained *a* : *b* : *c*, use the coordinates of a point on one of the lines to deduce the equation of the plane.

The standard of work was encouraging and can be improved even further if candidates can become more persistent in checking their work for arithmetic errors (particularly sign errors).

Answers: **(i)** 3**i** + **j** + **k**; **(ii)** 7*x* + *y* – 5*z* = 17.

Paper 9709/04

Paper 4

General comments

The early questions were well answered, most candidates obtaining a high proportion of the 20 marks available in **Questions 1** to **4**. Candidates found **Questions 5**, **6** and **7** more testing, but despite this some candidates scored full marks in these questions.

In **Question 1** candidates had no difficulty in identifying the forces acting horizontally, but in the different context of **Question 5** many candidates made mistakes in considering the forces acting horizontally on particle *A*.

In **Question 6** there was a reluctance among some candidates to accept the question as set, and a re-orientated diagram was copied on to the answer paper with the applied force of 5N acting vertically downwards.

In **Question 7** some candidates used the constant acceleration formulae inappropriately. This happened when one of the formulae was applied once only in respect of *A*'s motion over more than one of its separate stages. It also happened in respect of *B*'s motion for which the acceleration is not constant.

Comments on specific questions

Question 1

This question was very well attempted and many candidates scored all four marks. A candidate's error in part **(i)**, usually giving the answer as 50 N, did not necessarily preclude the scoring of all three marks in part **(ii)**.

Answers: **(i)** 320 N; **(ii)** 270 N.

Question 2

This was the best attempted question in the paper and many candidates scored full marks. The most common error occurred in part **(ii)**, in which some candidates brought forward $v = 20$ from part **(i)** and then obtained $t = 4.5$ s using $\frac{u+v}{2} = \frac{s}{t}$.

Answers: **(i)** 20 ms^{-1}; **(ii)** 3 s; **(iii)** 35 m.

Question 3

The most common error in part **(i)** of this question was to omit the factor cos15°, thus obtaining $WD = 25 \times 2 = 50$ J. Common mistakes in part **(ii)** included $N = 3g = 30$ N, $N = 25\sin 15° = 6.47$ N and $N = 3g + 6.5 = 36.5$ N.

Answers: **(i)** 48.3 J; **(ii)** 23.5 N.

Question 4

Very many candidates gave correct solutions in both parts of this question, using energy in part **(i)** and work and energy in part **(ii)**.

However some candidates obtained $h = 3.2$ m fortuitously in part **(i)** by effectively assuming that the path *AB* is a vertical straight line. Such candidates made inappropriate use of the formula $v^2 = u^2 + 2as$ with $a = g$.

When the erroneous assumption was carried through to part **(ii)** candidates found the vertical acceleration to be 4.5, from which the resistance was usually found as $0.15 \times 4.5 = 0.675$ N rather than $0.15(g - 4.5) = 0.825$ N.

Answers: **(i)** 3.2 m; **(ii)** 3.3 J.

Question 5

Most candidates obtained a correct equation by applying Newton's second law to *B*. However in applying Newton's second law to *A* many candidates included a term 4*g*, or excluded the frictional force of 0.6 N, or made both of these errors. Sign errors were also common.

The most common error in part **(ii)** was to use $a = g$, instead of the value of *a* obtained (or obtainable) from the simultaneous equations used in part **(i)**.

Answers: **(i)** 0.92 N; **(ii)** 1.2 ms^{-1}.

Question 6

This question proved to be the most difficult in the paper. Candidates who considered the equilibrium at *M* in part **(i)** were usually successful. However some thought the triangle of forces is similar to the triangle *AMB*, and had difficulty in relating the applied force of 5 N with any of the triangle's sides. Many other candidates used methods that involved the weight of *B*.

Errors in part **(ii)** were many and varied, including:

- taking *N* vertically upwards and *F* to be horizontal
- taking both *N* and *F* to be vertical
- taking *N* to be simply 0.2*g*
- taking *N* as 2*T*sin30°
- taking the weight to act horizontally
- taking *N* vertically and both *F* and the weight to act horizontally
- taking *F* along *BM*
- taking *F* along *BM* and *N* perpendicular to it
- having two vertical components of tension, sometimes both acting upwards and sometimes acting in opposite directions.

Candidates who failed to obtain $0.2g + F = T\cos 30°$ in part **(ii)** rarely made progress in part **(iii)**. Mistakes made in part **(iii)** included:

- replacing 0.2*g* in the above equation by *mg* or $(0.2g + m)$ instead of $(0.2 + m)g$
- failing to change the sign of *F* in the above equation, leading to $m = 0$
- changing $|F|$ from 2.33, in cases where *N* was taken as 0.2*g* in part **(ii)**, to '[candidate's μ] x $(0.2 + m)g$'.

Answers: **(ii)** 0.932; **(iii)** 0.466.

Question 7

Parts **(i)** and **(ii)** were very well attempted, most candidates scoring all five of the available marks.

Many candidates failed to score both marks in part **(iii)**. Common errors included:

- using $a(t) = v(t) \div t$ to find $a_B(t)$
- using $a_B(100)$ or $v_B(100) \div 100$ as the initial value of a_B.

Only the best candidates scored well in part **(iv)**. Errors included:

- failing to obtain $t = 250$
- using v_{max} x 500 for s_B
- obtaining the answer as $s_B(500) - s_A(500)$ or $s_B(300) - s_A(300)$ or $s_B(250) - s_A(300)$
- using 7.2 instead of 6.6 in $s_A(250) = 240 + \frac{1}{2}(4.8 + 6.6)150$
- omitting the 240 from $s_A(250) = 240 + \frac{1}{2}(4.8 + 6.6)150$
- failing to use integration for s_B.

Answers: **(i)** 2160 m; **(ii)** 0.048 ms^{-2}; **(iii)** 0.012 ms^{-2}; **(iv)** 155 m.

Paper 9709/05

Paper 5

General comments

Compared with last year, there was a much better response to this paper. With the possible exception of **Question 2**, many candidates of wide abilities found that they could make good inroads into all the questions.

On the whole, the solutions were well presented and in only an extremely small number of cases was there any evidence of candidates having insufficient time to complete the paper. One aspect of problem solving that could benefit candidates is the need to draw a *neat* sketch which contains all the relevant information, both known and that which is to be found. Hopefully this would then have avoided, for example, equating θ to the semi-vertical angle of the cone in **Question 2**. Or again, in **Question 6**, the component of the weight of the cyclist down the plane would not have been omitted so often when attempting to establish the differential equation.

Comments on specific questions

Question 1

The majority of candidates coped well with this straightforward example of circular motion and only the weakest failed to score maximum marks.

Answer: 25 000N.

Question 2

Despite the fact that the word 'cone' appeared four times in the question, many candidates took the centre of mass of the solid cone to be $\frac{20}{3}$ cm from the base. When candidates are provided with the formula list MF9, there can be no excuse for this sort of carelessness. Equally as bad were those less able candidates who apparently stumbled on the correct value for θ from $\tan\theta = \frac{20}{10}$. As mentioned above, this error could probably have been avoided if the sketch had not been so carelessly drawn.

What was expected in part **(ii)** was that candidates would establish the range of values of the coefficient of friction for which the cone would tilt before sliding. Many candidates merely stated on the first line of their solutions that $\mu > \tan\theta$ as though it was some quotable formula. Although a similar comment was made last year, it should be re-iterated that an inequality needs some qualifying statement. For example, it would have been equally true to state that $\mu < \tan\theta$ provided that there was the added statement 'the cone slides before tilting'.

Answers: **(i)** 63.4°; **(ii)** $\mu > 2$.

Question 3

Good candidates coped well with this question but many of the rest failed for a variety of reasons. In part **(i)** the compression of the spring was often taken to be 0.3 m rather than 0.1 m. It is perhaps also worth mentioning that confusion exists in the minds of some candidates between the modulus of elasticity associated with Hooke's Law and Young's modulus. In the application of Newton's Second Law of Motion the weight of the particle *P* was often omitted and the incorrect answer 110 ms^{-2} was seen all too often.

In part **(ii)** the E.P.E. was invariably found correctly but in part **(iii)** there was a lot of trouble experienced with the G.P.E., either through the incorrect value being used or even omitted altogether from the energy equation. Inevitably weak candidates tried to find the speed of *P* by using the formula $v^2 = u^2 + 2as$. This must be wrong because this formula can only be applied when the acceleration is constant. Here the force in the spring varies as the compression varies and hence the acceleration cannot be constant.

Answers: **(i)** 100 ms^{-2}; **(ii)** 1.1 J; **(iii)** 3 ms^{-1}.

Question 4

All candidates who had a good grasp of statistical ideas scored well on this question. In part **(i)**, although the obvious axis about which to take moments was *BC*, many chose an axis through *A* parallel to *BC*. There were often some tortuous methods to establish that the centre of mass of the triangle was 11.5 cm from *BC* but, nevertheless, a high proportion of candidates eventually arrived at the correct 6.37 cm. Most candidates appreciated that they had to take moments about *A* in part **(ii)** and to resolve vertically in part **(iii)**. Usually the less able candidates failed to appreciate that the tension could only be found by taking moments and the answer to part **(iii)** was invariably $T\sin 30°$.

Answers: **(ii)** 94.2 N; **(iii)** 32.9 N.

Question 5

Part **(i)** was well done. Although there were a number of ways of finding α, most candidates chose the simplest method by applying $v^2 = u^2 + 2as$ to the vertical component of the motion.

In part **(ii)** the response was disappointing in that candidates of all abilities made the mistake of assuming that the speed of the stone after rebounding was 10 ms^{-1}. The only possible conclusion that could be drawn was that many candidates labour under the impression that the speed of a projectile is constant at all points of its trajectory. Perhaps if more candidates had drawn a neat sketch with all information on it, instead of trying out all the projectile formulae that they knew, this error could have been avoided.

The ideas required to solve part **(iii)** were well known, although inevitably there were still some who attempted to find the angle using a ratio of displacements rather than speeds. A less obvious source of error was from those candidates who attempted to find the angle by adapting the Range formula. Although the horizontal displacement found in part **(ii)** was correctly doubled, the speed was taken to be 16 ms^{-1} rather than the speed with which the stone hits the ground ($\sqrt{208}$ ms^{-1}).

Answers: **(i)** 36.9°; **(ii)** 9.6 m; **(iii)** 56.3°.

Question 6

There was a high degree of success with parts **(i)** and **(ii)**. Even though the required answers were given, many candidates handled the application of Newton's Second Law of Motion in part **(i)** and the integration and algebraic manipulation in part **(ii)** in a confident manner. The most frequent errors in part **(ii)** were the omission of the minus sign in the integration of $\frac{1}{5-v}$ and the lack of a constant of integration (or the blithe assumption that putting $t = 0$ and $v = 0$ must lead to $c = 0$).

Only the best candidates made a success of part **(iii)** by realising that further integration was necessary by putting $v = \frac{ds}{dt}$. A few chose the harder route by making a fresh start with the original differential equation with acceleration = $v\frac{dv}{dx}$. Although the candidates knew what to do, the solutions often foundered on the inability to integrate correctly. All other attempts seemed to be based on finding the speed at the top of the slope (4.32 ms^{-1}) and then erroneously applying a constant acceleration formula (e.g. $s = \frac{1}{2}(0 + 4.32)20$). Again, as in **Question 3 (iii)**, as there is a variable force ($8v$ N), this must lead to a variable acceleration.

Answer: **(iii)** 56.8 m.

Paper 9709/06

Paper 6

General comments

This paper produced a wide range of marks. Most candidates had covered the syllabus adequately with only a few Centres gaining consistently low marks. Premature approximation leading to a loss of marks was only witnessed in a few scripts, most candidates realising the necessity of working with four significant figures. One unforeseen problem was the candidates' failure to appreciate the difference between decimal places and significant figures. This was particularly noticeable in answers such as 0.0419 and 0.0451, where many gave answers as 0.042 etc. Candidates seemed to have sufficient time to answer all the questions, and only the weaker candidates answered questions out of order.

Comments on specific questions

Question 1

This question caused problems for many candidates. Many candidates confused this with a binomial situation and tried to find an '*n*' and a '*p*'. Others found f^2x for the variance, and as usual, some candidates found the standard deviation.

Answers: 0.850, 0.978.

Question 2

Approximately half the candidates appreciated the need to find a scaled frequency, or frequency density. It was pleasing to see most candidates had touching bars on the histogram, with the vertical axis labelled as frequency density, but only a small number labelled the horizontal axis as being area or m^2 and thus many candidates lost a mark.

Question 3

Apart from a few Centres where the normal distribution did not appear to have been taught with any rigour, this question was well done with most candidates finding an appropriate *z*-value. The range of *z*-values was wider than expected, with many ranging from 0.492 to 0.5 for $\Phi(0.69)$. Only values of 0.495 and 0.496 were accepted. Solving the simultaneous equations was well done and almost all candidates who had done some work on the normal distribution scored at least 4 marks out of 6.

Answers: 8.91, 23.6.

Question 4

This question was the worst attempted on the paper. A tree diagram would perhaps have helped. Many candidates wrote $\frac{4}{5}$ instead of $\frac{19}{20}$, many thought it was a 'without replacement' type of question and many misunderstood the last part as meaning 'completes his collection with less than 3 pictures'.

Answers: **(i)** 0.774; **(ii)** 0.204; **(iii)** 0.0451.

Question 5

The tree diagram was well drawn by the majority of candidates. A few failed to realise that the male/female branch had to come first, and many multiplied their probabilities together before writing the second probabilities on the tree diagram, and then proceeded to multiply a third time. A minority of the candidates appreciated that this was a conditional probability question and thus many scored only 2 marks out of 6.

Answer: 0.746.

Question 6

This permutation and combination question was very well attempted by almost all candidates, many picking up 4 or more marks. Sometimes the answers were not integers, and occasionally they became probabilities. The answers were not always fully correct, but there were signs of sensible reasoning.

Answers: **(a)(i)** 18 564, **(ii)** 6188; **(b)(i)** 40 320, **(ii)** 2880.

Question 7

This very straightforward normal distribution first part gained nearly full marks for everyone who had studied the subject. However, quite a few lost the final mark for this part because of incorrect use of the four-figure Normal tables. The second part was a binomial situation based on the first part, the answer of which had already been calculated. Almost without exception, candidates calculated the probability all over again, suggesting they had not appreciated the significance of what they were doing in part **(i)**. The answers to part **(iii)** were almost all wrong. Candidates clearly did not appreciate the difference between 'mean' and 'median'. Neither did they realise that a normal distribution is symmetric with the mean and median coinciding.

Answers: **(i)** 0.3735 (0.374); **(ii)** 0.0419; **(iii)** box plot is skew, not symmetric so not normal.

Question 8

This question was the easiest question by far and was well done by a large majority. For some it provided half their marks. Rounding errors and premature approximation led to a few marks being lost, and not everyone realised that part **(iii)** entailed adding probabilities for two discrete numbers.

Answers: **(i)** $\frac{1}{18}$ or 0.0556; **(ii)** 2.78, 1.17; **(iii)** 0.611.

Paper 9709/07

Paper 7

General comments

This was a well attempted paper where most candidates were able to apply their knowledge of the subject. There was no evidence of any time pressure on candidates to complete the paper and, on the whole, presentation was of an acceptable standard. Once again some candidates lost accuracy marks by writing down final answers to two significant figures, instead of three, and in some cases did not appreciate the difference between three significant figures and three decimal places. **Question 4** was particularly well answered, while **Questions 6** and **7** proved to be the most demanding. There were cases of particularly good scripts with candidates gaining full marks, but equally some very poor attempts were also seen. A good spread of marks was obtained.

Comments on specific questions

Question 1

This question was reasonably well attempted, though some candidates did not appreciate that the width was $2 \times z \times$ s.e. and were therefore unable to make any progress with the question. Errors included using $z = 1.645$ rather than $z = 1.96$ and more commonly omitting the factor of 2 on the width (that is, using the inequality $z \times$ s.e. < 2).

Answer: $n = 14$.

Question 2

A Poisson approximation was required for this question. Many candidates used a normal approximation which was not valid since $np < 5$. Also some candidates ignored the instruction to use an approximation and used Bin(45000, 0.0001). Some marks were available for these candidates but full marks were only awarded for using the correct Poisson approximation (even though the same final answer could have been obtained). Candidates who correctly used Po(4.5) generally reached the correct final answer. Errors such as Po(0.45) or Po(0.22) were seen as well as choosing the wrong probabilities to sum. It was also noted that some candidates failed to *add* their probabilities of 2, 3, and 4 and even P(2) $\times$ P(3) + P(4) was seen.

Answer: 0.471.

Question 3

Most candidates were able to score marks on this question. However, many errors were seen in attempting to find the correct mean (19) and variance (12) of Su Chen's upgraded throw. Use of N(19,17) was common.

Answer: 0.586.

Question 4

This was a particularly well attempted question, even by weaker candidates. One error frequently seen was to miscalculate l and use 2.5 rather than 0.25. A final answer of 0.002 (or 0.0022) was very common and showed a lack of understanding of three significant figures. In part **(ii)** some candidates used $e^{-k} = 0.9$ instead of $e^{-k/80} = 0.9$, but many candidates successfully found the correct value of k. Again 8.4 rather 8.43 was often given as the final answer and without the previous unrounded figure accuracy marks were lost. It was surprising on this question that a few (even good) candidates used log rather than ln, even stating log e = 1.

Answers: **(i)** 0.00216; **(ii)** 8.43.

Question 5

This was also a reasonably well attempted question. Some candidates used 117 rather than the s.e. of $\frac{117}{\sqrt{26}}$, and a common error in part **(ii)** was to use a one-tail test (though follow through marks were available). It was pleasing to note that, on the whole, candidates stated their null and alternative hypothesis and were able to give final conclusions related to the situation in the question. It is important that candidates show that they are *comparing* their value with $\pm$ 1.645 (or equivalent), either by an inequality statement or a clear diagram. Some candidates failed to show this comparison and consequently marks were lost.

Answers: **(i)** 0.985; **(ii)** No significant change.

Question 6

Candidates were particularly good at part **(i)** where they were required to define type I and type II errors. However, despite knowing the definition very few candidates were able to apply this knowledge in part **(ii)**. The situation required Bin(5, 0.94) for part **(a)** and Bin(5, 0.7) for part **(b)**. Unfortunately very few candidates used these distributions with the correct parameters and attempts at other Binomials, or a Normal, or even a Poisson distribution were seen. This was consequently a low scoring question; with full marks only occasionally seen.

Answers: **(i)(a)** Rejecting H_0 when it is true, **(b)** Accepting H_0 when it is false; **(ii)(a)** 0.266, **(b)** 0.168.

Question 7

This was, surprisingly, not a particularly well attempted question, though many candidates made a good attempt at integrating by parts in **(iii)**.

Part **(i)** required the candidates to show that $k = 3$, and many errors and unconvincing solutions were seen. An integral from zero to infinity of ke^{-3x} was required and should have been equated to one. Many candidates were unable to state these limits, and integrals with no limits or incorrect ones (1 to 2 or 0 to 1) were common. Full, convincing, working was required for part **(i)**.

Part **(ii)** produced better solutions though sign mistakes were common. Integrals with incorrect limits from 0 to $\frac{1}{4}$ were also seen.

In part **(iii)** many candidates gained a few marks for attempting to integrate by parts. Limits of zero to infinity were needed and many candidates did not use these and made similar errors to those in part **(i)**. Again, sign mistakes were common.

Weaker candidates confused mean with median.

Answers: **(ii)** 0.0959; **(iii)** $\frac{1}{3}$.