

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

## MATHEMATICS

9709/52
May/June 2012
1 hour 15 minutes

Additional Materials: Answer Booklet/Paper Graph Paper List of Formulae (MF9)

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10 \mathrm{~m} \mathrm{~s}^{-2}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50 .
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1 A particle $P$ of mass 0.6 kg is projected horizontally with velocity $2 \mathrm{~m} \mathrm{~s}^{-1}$ from a point $O$ on a smooth horizontal surface. A horizontal force of magnitude $0.3 x \mathrm{~N}$ acts on $P$ in the direction $O P$, where $x \mathrm{~m}$ is the distance of $P$ from $O$. Calculate the velocity of $P$ when $x=8$.

2 A uniform hemispherical shell of weight 8 N and a uniform solid hemisphere of weight 12 N are joined along their circumferences to form a non-uniform sphere of radius 0.2 m .
(i) Show that the distance between the centre of mass of the sphere and the centre of the sphere is 0.005 m .

This sphere is placed on a horizontal surface with its axis of symmetry horizontal. The equilibrium of the sphere is maintained by a force of magnitude $F \mathrm{~N}$ acting parallel to the axis of symmetry applied to the highest point of the sphere.
(ii) Calculate $F$.

3 A light elastic string has natural length 2.2 m and modulus of elasticity 14.3 N . A particle $P$ of mass $m \mathrm{~kg}$ is attached to the mid-point of the string. The ends of the string are attached to fixed points $A$ and $B$ which are 2.4 m apart at the same horizontal level. $P$ is released from rest at the mid-point of $A B$. In the subsequent motion $P$ has its greatest speed at a point 0.5 m below $A B$.
(i) Find $m$.
(ii) Calculate the greatest speed of $P$.

4 A particle $P$ of mass 0.25 kg moves in a straight line on a smooth horizontal surface. At time $t \mathrm{~s}$ the velocity of $P$ is $v \mathrm{~m} \mathrm{~s}^{-1}$. A variable force of magnitude $3 t \mathrm{~N}$ opposes the motion of $P$.
(i) Given that $P$ comes to rest when $t=3$, find $v$ when $t=0$.
(ii) Calculate the distance travelled by $P$ in the interval $0 \leqslant t \leqslant 3$.

5 A ball is projected with velocity $25 \mathrm{~m} \mathrm{~s}^{-1}$ at an angle of $70^{\circ}$ above the horizontal from a point $O$ on horizontal ground. The ball subsequently bounces once on the ground at a point $P$ before landing at a point $Q$ where it remains at rest. The distance $P Q$ is 17.1 m .
(i) Calculate the time taken by the ball to travel from $O$ to $P$ and the distance $O P$.
(ii) Given that the horizontal component of the velocity of the ball does not change at $P$, calculate the speed of the ball when it leaves $P$.

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The diagram shows a uniform lamina $A B C D E F$, formed from a semicircle with centre $O$ and radius 1 m by removing a semicircular part with centre $O$ and radius $r \mathrm{~m}$.
(i) Show that the distance in metres of the centre of mass of the lamina from $O$ is

$$
\begin{equation*}
\frac{4\left(1+r+r^{2}\right)}{3 \pi(1+r)} \tag{4}
\end{equation*}
$$

The centre of mass of the lamina lies on the arc $A B C$.
(ii) Show that $r=0.494$, correct to 3 significant figures.

The lamina is freely suspended at $F$ and hangs in equilibrium.
(iii) Find the angle between the diameter of the lamina and the vertical.

7 Particles $P$ and $Q$, of masses 0.8 kg and 0.5 kg respectively, are attached to the ends of a light inextensible string which passes through a small hole in a smooth horizontal table of negligible thickness. $P$ moves with constant angular speed $6.25 \mathrm{rad} \mathrm{s}^{-1}$ in a circular path on the surface of the table.
(i) It is given that $Q$ is stationary and that the part of string attached to $Q$ is vertical. Calculate the radius of the path of $P$, and find the speed of $P$.
(ii) It is given instead that the part of string attached to $Q$ is inclined at $60^{\circ}$ to the vertical, and that $Q$ moves in a horizontal circular path below the table, also with constant angular speed $6.25 \mathrm{rad} \mathrm{s}^{-1}$. Calculate the total length of the string.

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