UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2010 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/33 Paper 33, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2010 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	*EITHER*: State or imply non-modular inequality $(x-3)^2 > (2(x+1))^2$, or corresponding quadratic equation, or pair of linear equations $(x-3) = \pm 2(x+1)$		B1	
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations		M1	
	Obtain critical values -5 and $\frac{1}{3}$		A1	
	State answer $-5 < x < \frac{1}{3}$		A1	
	OR: Obtain the critical value $x = -5$ from a graphical method, or by inspection, or by solving a linear equation or inequality		B1	
	Obtain the critical value $x = \frac{1}{3}$ similarly		B2	
	State answer $-5 < x < \frac{1}{3}$		B1	[4]
	[Do not condone $\le$ for $<$; accept 0.33 for $\frac{1}{3}$.]			
2	**(i)**	State or imply $3\ln y = \ln A + 2x$ at any stage	B1	
		State gradient is $\dfrac{2}{3}$, or equivalent	B1	[2]
	(ii)	Substitute $x = 0$, $\ln y = 0.5$ and solve for A	M1	
		Obtain $A = 4.48$	A1	[2]
3	Attempt to use $\tan(A \pm B)$ formula and obtain an equation in $\tan x$		M1	
	Obtain 3-term quadratic $2\tan^2 x + 3\tan x - 1 = 0$, or equivalent		A1	
	Solve a 3-term quadratic and find a numerical value of x		M1	
	Obtain answer 15.7°		A1	
	Obtain answer 119.3° and no others in the given interval		A1	[5]
	[Ignore answers outside the given interval. Treat answers in radians, 0.274 and 2.08, as a misread.]			
4	Separate variables correctly		B1	
	Obtain term $k\ln(4-x^2)$, or terms $k_1\ln(2-x) + k_2\ln(2+x)$		B1	
	Obtain term $-2\ln(4-x^2)$, or $-2\ln(2-x) - 2\ln(2+x)$, or equivalent		B1	
	Obtain term t, or equivalent		B1	
	Evaluate a constant or use limits $x = 1$, $t = 0$ in a solution containing terms $a\ln(4-x^2)$ and bt or terms $c\ln(2-x)$, $d\ln(2+x)$ and bt		M1	
	Obtain correct solution in any form, e.g. $-2\ln(4-x^2) = t - 2\ln 3$		A1	
	Rearrange and obtain $x^2 = 4 - 3\exp(-\frac{1}{2}t)$, or equivalent (allow use of $2\ln 3 = 2.20$)		A1	[7]
5	**(i)**	State derivative $-e^{-x} - (-2)e^{-2x}$, or equivalent	B1 + B1	
		Equate derivative to zero and solve for x	M1	
		Obtain $p = \ln 2$, or exact equivalent	A1	[4]
	(ii)	State indefinite integral $-e^{-x} - (-\frac{1}{2})e^{-2x}$, or equivalent	B1 + B1	
		Substitute limits $x = 0$ and $x = p$ correctly	M1	
		Obtain given answer following full and correct working	A1	[4]

6	**(i)**	Use correct quotient or product rule	M1	
		Obtain correct derivative in any form, e.g. $\dfrac{1}{x(x+1)} - \dfrac{\ln x}{(x+1)^2}$	A1	
		Equate derivative to zero and obtain the given equation correctly	A1	
		Consider the sign of $x - \dfrac{(x+1)}{\ln x}$ at $x = 3$ and $x = 4$, or equivalent	M1	
		Complete the argument with correct calculated values	A1	[5]
	(ii)	Use the iterative formula correctly at least once, using or reaching a value in the interval (3, 4)	M1	
		Obtain final answer 3.59	A1	
		Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (3.585, 3.595)	A1	[3]
7	**(i)**	Use correct cos(*A* + *B*) formula to express $\cos 3\theta$ in terms of trig functions of 2θ and θ	M1	
		Use correct trig formulae and Pythagoras to express $\cos 3\theta$ in terms of $\cos\theta$	M1	
		Obtain a correct expression in terms of $\cos\theta$ in any form	A1	
		Obtain the given identity correctly	A1	[4]
		[SR: Give M1 for using correct formulae to express RHS in terms of $\cos\theta$ and $\cos 2\theta$, then M1A1 for expressing in terms of either only $\cos 3\theta$ and $\cos\theta$, or only $\cos 2\theta$, $\sin 2\theta$, $\cos\theta$, and $\sin\theta$, and A1 for obtaining the given identity correctly.]		
	(ii)	Use identity and integrate, obtaining terms $\frac{1}{4}(\frac{1}{3}\sin 3\theta)$ and $\frac{1}{4}(3\sin\theta)$, or equivalent	B1 + B1	
		Use limits correctly in an integral of the form $k\sin 3\theta + l\sin\theta$	M1	
		Obtain answer $\dfrac{2}{3} - \dfrac{3}{8}\sqrt{3}$, or any exact equivalent	A1	[4]
8	**(a)**	*EITHER*: Substitute $1 + i\sqrt{3}$, attempt complete expansions of the x^3 and x^2 terms	M1	
		Use $i^2 = -1$ correctly at least once	B1	
		Complete the verification correctly	A1	
		State that the other root is $1 - i\sqrt{3}$	B1	
		*OR*1: State that the other root is $1 - i\sqrt{3}$	B1	
		State quadratic factor $x^2 - 2x + 4$	B1	
		Divide cubic by 3-term quadratic reaching partial quotient $2x + k$	M1	
		Complete the division obtaining zero remainder	A1	
		*OR*2: State factorisation $(2x+3)(x^2 - 2x + 4)$, or equivalent	B1	
		Make reasonable solution attempt at a 3-term quadratic and use $i^2 = -1$	M1	
		Obtain the root $1 + i\sqrt{3}$	A1	
		State that the other root is $1 - i\sqrt{3}$	B1	[4]
	(b)	Show point representing $1 + i\sqrt{3}$ in relatively correct position on an Argand diagram	B1	
		Show circle with centre at $1 + i\sqrt{3}$ and radius 1	B1√	
		Show line for $\arg z = \frac{1}{3}\pi$ making $\frac{1}{3}\pi$ with the real axis	B1	
		Show line from origin passing through centre of circle, or the diameter which would contain the origin if produced	B1	
		Shade the relevant region	B1√	[5]

9 **(i)** State or imply partial fractions of the form $\dfrac{A}{1-2x}+\dfrac{B}{2+x}+\dfrac{C}{(2+x)^2}$ B1

Use any relevant method to determine a constant M1
Obtain one of the values $A = 1$, $B = 1$, $C = -2$ A1
Obtain a second value A1
Obtain the third value A1 [5]

[The form $\dfrac{A}{1-2x}+\dfrac{Dx+E}{(2+x)^2}$, where $A = 1$, $D = 1$, $E = 0$, is acceptable scoring B1M1A1A1A1 as above.]

(ii) Use correct method to obtain the first two terms of the expansion of $(1-2x)^{-1}$, $(2+x)^{-1}$, $(2+x)^{-2}$, $(1+\frac{1}{2}x)^{-1}$, or $(1+\frac{1}{2}x)^{-2}$ M1

Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction A1√ + A1√ + A1√

Obtain answer $1+\dfrac{9}{4}x+\dfrac{15}{4}x^2$, or equivalent A1 [5]

[Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$, are not sufficient for the M1. The f.t. is on A, B, C.]

[For the A, D, E form of partial fractions, give M1A1√A1√ for the expansions then, if $D \neq 0$, M1 for multiplying out fully and A1 for the final answer.]

[In the case of an attempt to expand $(4+5x-x^2)(1-2x)^{-1}(2+x)^{-2}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]

[SR: If B or C omitted from the form of fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**.]

[SR: If D or E omitted from the form of fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**.]

10 **(i)** Express general point of the line in component form, e.g. $(2+\lambda, -1+2\lambda, -4+2\lambda)$ B1
Substitute in plane equation and solve for λ M1
Obtain position vector $4\mathbf{i} + 3\mathbf{j}$, or equivalent A1 [3]

(ii) State or imply a correct vector normal to the plane, e.g. $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ B1
Using the correct process, evaluate the scalar product of a direction vector for l and a normal for p M1
Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine or inverse sine of the result M1
Obtain answer 26.5° (or 0.462 radians) A1 [4]

(iii) *EITHER*: State $a + 2b + 2c = 0$ or $3a - b + 2c = 0$ B1
Obtain two relevant equations and solve for one ratio, e.g. $a : b$ M1
Obtain $a : b : c = 6 : 4 : -7$, or equivalent A1
Substitute coordinates of a relevant point in $6x + 4y - 7z = d$ and evaluate d M1
Obtain answer $6x + 4y - 7z = 36$, or equivalent A1

*OR*1: Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) \times (3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ M1
Obtain two correct components of the product A1
Obtain correct product, e.g. $6\mathbf{i} + 4\mathbf{j} - 7\mathbf{k}$ A1
Substitute coordinates of a relevant point in $6x + 4y - 7z = d$ and evaluate d M1
Obtain answer $6x + 4y - 7z = 36$, or equivalent A1

*OR*2: Attempt to form 2-parameter equation with relevant vectors M1
State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{j} - 4\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ A1
State three equations in x, y, z, λ, μ A1
Eliminate λ and μ M1
Obtain answer $6x + 4y - 7z = 36$, or equivalent A1 [5]