UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2010 question paper

for the guidance of teachers

	9709 MATHEMATICS
9709/13	Paper 13, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

<u>Penalties</u>

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 **(i)** $a = 12$, $ar = -6 \rightarrow r = -\frac{1}{2}$	M1	Attempt at r from "ar"
$ar^9 = \dfrac{-3}{128}$	M1 A1 [3]	ar^9 must be correct. co
(ii) $S_\infty = \dfrac{a}{1-r}$ used $\rightarrow 8$	M1 A1 [2]	Correct formula used. M1 needs $\lvert r \rvert < 1$
2 **(i)** $\left(x - \dfrac{2}{x}\right)^6 = x^6 - 12x^4 + 60x^2$	B1 ×3 [3]	co
(ii) $\times (1 + x^2) \rightarrow 60 - 12 = 48$	M1 A1√ [2]	Must be exactly 2 terms. √ from his **(i)**.
3 $\mathrm{f}: x \mapsto a + b\cos x$		
(i) $\mathrm{f}(0) = 10$, $a + b = 10$		
$\mathrm{f}(^2/_3\pi) = 1$, $a - \dfrac{b}{2} = 1$	B1	EITHER OF THESE
$\rightarrow a = 4$, $b = 6$	B1 [2]	both co
(ii) Range is –2 to 10.	B1√ [1]	√ for his "$a - b$" to "$a + b$"
(iii) $\cos\left(\dfrac{5}{6}\pi\right) = -\cos\left(\dfrac{1}{6}\pi\right) = -\dfrac{\sqrt{3}}{2}$	B1	For $\cos 30° = \frac{1}{2}\sqrt{3}$ used somewhere.
$\rightarrow 4 - 3\sqrt{3}$	B1 [2]	co
4 **(i)** $2\sin x \tan x + 3 = 0$		
$2\sin x \dfrac{\sin x}{\cos x} + 3 = 0$	M1	For using tan = sin ÷ cos
$2\dfrac{(1 - \cos^2 x)}{\cos x} + 3 = 0$	M1	For using $\sin^2 + \cos^2 = 1$ <u>and</u> everything correct
$\rightarrow 2\cos^2 x - 3\cos x - 2 = 0$	[2]	Answer given – check.
(ii) $2\cos^2 x - 3\cos x - 2 = 0$ $\rightarrow \cos x = -\frac{1}{2}$ or 2 $x = 120°$ or $240°$	M1 A1 B1√ [3]	Solution of quadratic. co. √ for 360 – his answer.

5 $\frac{dy}{dx} = \frac{6}{\sqrt{3x-2}}$		
(i) $x = 2$, tangent has gradient 3	M1	Use of $m_1m_2 = -1$ with dy/dx
$\rightarrow$ normal has gradient $-\frac{1}{3}$	M1 A1	Correct form of line eqn. for normal
$\rightarrow y - 11 = -\frac{1}{3}(x-2)$	[3]	
(ii) Integrate $\rightarrow 6\frac{\sqrt{3x-2}}{\frac{1}{2}} \div 3$	B1 B1	Without the ÷3 For ÷3, even if B0 above
$\rightarrow y = 4\sqrt{3x-2} + c$ through (2,11)	M1	Using (2, 11) for c
$\rightarrow y = 4\sqrt{3x-2} + 3$	A1 [4]	co
6 $\overrightarrow{OA} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$, $\overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} + 8\mathbf{k}$, $\overrightarrow{OC} = -\mathbf{i} - 2\mathbf{j} + 10\mathbf{k}$		
(i) (±) $2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$ (±) $4\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$	B1 B1	co co
$\overrightarrow{AB}.\overrightarrow{CB} = 16$	M1	Needs to be scalar.
$\overrightarrow{AB}.\overrightarrow{CB} = \sqrt{36}\sqrt{36}\cos\theta$	M1	For product of 2 moduli and cosine
$\theta = 63.6°$	M1 A1 [6]	All correct.
(ii) Perimeter $= 6 + 6 + \sqrt{40}$ or $6 + 6 + 6\sin 31.8° \times 2$	M1	Correct overall method for perimeter.
$\rightarrow 18.32$	A1 [2]	co
7 **(i)** $\sin \frac{1}{2}\theta = \frac{6}{10}$	M1	Use of trig with/without radians
Angle DOE = 1.287 radians.	A1 [2]	co – answer given.
(ii) P = 12 + 12 + 2 × 10 × angle BOD Angle BOD = (π – 1.287) → 61.1	M1 M1 A1 [3]	Use of $s = r\theta$ for arc length. Correct angle co
(iii) Sector DOE = ½ × 10^2 × 1.287 Triangle DOE = ½ × 10^2 × sin 1.287 Area = π × 10^2 – (2 sectors – 2 triangles) (or 48 + 48 + 2×½×10^2×(π – 1.287) M1 M1	M1 M1	Correct formula used with radians. Correct formula used with radians.
→ 281 <u>or</u> 282	A1 [3]	co

8 **(i)** Mid-point of $AC = (2, 3)$ Gradient of $AC = {}^1/_3$ Gradient of $BD = -3$ Equation $y - 3 = -3(x - 2)$	B1 M1 A1 [3]	Co Use of $m_1m_2 = -1$ Co
(ii) If $x = 0$, $y = 9$, B (0, 9) Vector move D (4, –3)	B1√ M1 A1 [3]	√ on his equation. Valid method. co.
(iii) $AC = \sqrt{40}$ $BD = \sqrt{160}$ Area = 40 (or by matrix method M2 A1)	 M1 M1 A1 [3]	 Correct use on either AC or BD, Full and correct method. co
9 $y = x + \dfrac{4}{x}$		
(i) $x + \dfrac{4}{x} = 5 \rightarrow A\ (1, 5), B(4, 5)$	B1 B1	co. co.
$\dfrac{dy}{dx} = 1 - \dfrac{4}{x^2}$	M1	Differentiates.
$= 0$ when $x = 2$, M (2, 4).	DM1 A1 [5]	Setting to 0. co.
(ii) Vol of cylinder $= \pi 5^2.3$ Vol under curve $= \pi \int y^2\ dx$	B1 M1	Any valid method. Attempt at integrating y^2
Integral $= \dfrac{x^3}{3} - \dfrac{16}{x} + 8x$	A2, 1, 0	Allow if no π present.
Uses his limits "1 to 4" $\rightarrow 75\pi - 57\pi = 18\pi$	DM1 A1 [6]	Using his limits. co.
10 $f : x \mapsto 2x^2 - 8x + 14$		
(i) $y + kx = 12$, Sim Eqns. $\rightarrow 2x^2 - 8x + kx + 2 = 0$ Use of $b^2 - 4ac$ $\rightarrow (k - 8)^2 = 16 \rightarrow k = 12$ or 4.	M1 A1 M1 A1 [4]	Complete elimination of y (or x) Uses $b^2 - 4ac$ on eqn = 0, no "x" in a, b, c. co.co
(ii) $2x^2 - 8x + 14 = 2(x - 2)^2 + 6$	B1×3 [3]	
(iii) Range of f $\geqslant 6$.	B1√ [1]	√ for c or from calculus.
(iv) Smallest $A = 2$	B1√ [1]	√ to answer to **(ii)**.
(v) Makes x the subject Order of operations correct.	M1 M1	Could interchange x, y first. Order must be correct.
$g^{-1}(x) = \sqrt{\dfrac{x-6}{2}} + 2$	A1 [3]	co