UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2009 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/03 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2009 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 State or imply $2 + e^{-x} = e^2$ B1

Carry out method for finding $\pm x$ from $e^{\pm x} = k$, where $k > 0$, following sound ln or exp work M1

Obtain $x = -\ln(e^2 - 2)$, or equivalent expression for x A1

Obtain answer $x = -1.68$ A1 **4**

[The answer must be given to 2 decimal places]

[SR: the M1 is available for attempts starting with $2 + e^{-x} = 10^2$]

2 **(i)** State or imply 3 of the 4 ordinates 1, 1.069389..., 1.290994..., 1.732050... B1

Use correct formula, or equivalent, with $h = \frac{1}{12}\pi$ and four ordinates M1

Obtain answer 0.98 with no errors seen A1 **3**

[Accept $h = 0.26$ but not $h = 15$ when awarding the M1]

[SR: if only $\sqrt{\frac{5}{3}}$ and/or $\sqrt{3}$ are given, and decimals are not seen, the B1 is available]

[SR: solutions with 2 or 4 intervals can score only the M1 for a correct expression]

(ii) Justify statement that the second estimate would be less than E B1 **1**

3 **(i)** Use $\cot A = 1/\tan A$ or $\cos A/\sin A$ and/or $\operatorname{cosec} A = 1/\sin A$ on at least two terms M1

Use a correct double angle formula or the $\sin(A - B)$ formula at least once M1

Obtain given result A1 **3**

(ii) Solve $\cot\theta = 2$ for θ and obtain answer 26.6° B1

Obtain answer 206.6° and no others in the given range B1√ **2**

[Ignore answers outside the given range. Treat answers given in radians as a misread]

4 **(i)** Compare signs of $x^3 - 2x - 2$ when $x = 1$ and $x = 2$, or equivalent M1

Complete the argument with correct calculations A1 **2**

(ii) State or imply the equation $x = (2x^3 + 2) / (3x^2 - 2)$ B1

Rearrange this in the form $x^3 - 2x - 2 = 0$, or work *vice versa* B1 **2**

(iii) Use the iterative formula correctly at least once with $x_n > 0$ M1

Obtain final answer 1.77 A1

Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change

In the interval (1.765, 1.775) A1 **3**

5 **(i)** State correct first two terms of the expansion of $(1+ax)^{\frac{2}{3}}$, i.e. $1+\frac{2}{3}ax$ B1

Form an expression for the coefficient of x in the expansion of $(1+2x)(1+ax)^{\frac{2}{3}}$ and equate it to zero M1

Obtain $a = -3$ A1 **3**

(ii) Obtain correct unsimplified terms in x^2 and x^3 in the expansion of $(1-3x)^{\frac{2}{3}}$ or $(1+ax)^{\frac{2}{3}}$ B1√ + B1√

Carry out multiplication by $1 + 2x$ obtaining two terms in x^3 M1

Obtain final answer $-\frac{10}{3}x^3$, or equivalent A1 **4**

[Symbolic binomial coefficients, e.g. $\binom{\frac{2}{3}}{1}$, are not acceptable for the B marks in **(i)** or **(ii)**]

6 **(i)** **EITHER** State $\frac{dx}{dt} = -3a\cos^2 t \sin t$ or $\frac{dy}{dt} = 3a\sin^2 t \cos t$, or equivalent B1

Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ M1

OR State $\frac{2}{3}x^{-\frac{1}{3}}dx$ or $\frac{2}{3}y^{-\frac{1}{3}}dy$ as differentials of $x^{\frac{2}{3}}$ or $y^{\frac{2}{3}}$ respectively, or equivalent B1

Obtain $\frac{dy}{dx}$ in terms of t, having taken the differential of a constant to be zero M1

Obtain $\frac{dy}{dx}$ in any correct form A1 **3**

(ii) Form the equation of the tangent M1

Obtain the equation in any correct form A1

Obtain the given answer A1 **3**

(iii) State the x-coordinate of X or the y-coordinate of Y in any correct form B1

Obtain the given answer with no errors seen B1 **2**

7 **(i)** Use quadratic formula, or completing the square, or the substitution $z = x + iy$ to find a root, using $i^2 = -1$ M1

Obtain a root, e.g. $1 - \sqrt{3}i$ A1

Obtain the other root, e.g. $-1 - \sqrt{3}i$ A1 **3**

(ii) Represent both roots on an Argand diagram in relatively correct positions B1√ **1**

(iii) State modulus of both roots is 2 B1√

State argument of $1 - \sqrt{3}i$ is $-60°$ (or $300°$, $-\frac{1}{3}\pi$, $-\frac{5}{3}\pi$) B1√

State argument of $-1 - \sqrt{3}i$ is $-120°$ (or $240°$, $-\frac{2}{3}\pi$, $-\frac{4}{3}\pi$) B1√ **3**

(iv) Give a complete justification of the statement B1 **1**

[The A marks in **(i)** are for the final versions of the roots. Allow $(\pm 2 - 2\sqrt{3}i)/2$ as final answer. The remaining marks are only available for roots such that $xy \neq 0$.]

[Treat answers to **(iii)** in polar form as a misread]

8 **(i)** State or imply the form $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{10 - x}$ B1

Use any relevant method to determine a constant M1

Obtain one of the values $A = 1$, $B = 10$, $C = 1$ A1

Obtain the remaining two values A1 **4**

[The form $\frac{Dx + E}{x^2} + \frac{C}{10 - x}$ is acceptable and leads to $D = 1$, $E = 10$, $C = 1$]

(ii) Separate variables and attempt integration of both sides M1

Obtain terms $\ln x$, $-10/x$, $-\ln(10 - x)$, or equivalent A1√ + A1√ + A1√

Evaluate a constant or use limits $x = 1$, $t = 0$ with a solution containing 3 of the terms $k\ln x$, l/x, $m\ln(10 - x)$ and t, or equivalent M1

Obtain any correct expression for t, e.g. $t = \ln\left(\frac{9x}{10 - x}\right) - \frac{10}{x} + 10$ A1 **6**

[A separation of the form $\frac{a\mathrm{d}x}{x^2(10 - x)} = b\mathrm{d}t$ is essential for the M1. The f.t. is on A, B, C]

[If A or B (D or E) omitted from the form of fractions, give B0M1A0A0 in **(i)**; M1A1√ A1√M1A0 in **(ii)**]

9 (i) EITHER Substitute coordinates of general point of l in equation of plane and equate constant terms, obtaining an equation in b and c — M1 *
Obtain a correct equation, e.g. $8 + 2b - c = 1$ — A1
Equate the coefficient of t to zero, obtaining an equation in b and c — M1 *
Obtain a correct equation, e.g. $4 - b - 2c = 0$ — A1

OR Substitute (4, 2, –1) in the plane equation — M1 *
Obtain a correct equation in b and c, e.g. $2b - c = -7$ — A1
EITHER Find a second point on l and obtain an equation in b and c — M1 *
Obtain a correct equation in b and c, e.g. $b + 2c = 4$ — A1
OR Calculate scalar product of a direction vector for l and a vector normal for the plane and equate to zero — M1 *
Obtain a correct equation for b and c — A1
Solve for b or for c — M1(dep*)
Obtain $b = -2$ and $c = 3$ — A1 **6**

(ii) EITHER Find $\overrightarrow{PQ}$ for a point Q on l with parameter t, e.g. $4\mathbf{i} - 5\mathbf{k} + t(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ — B1
Calculate scalar product of $\overrightarrow{PQ}$ and a direction vector for l and equate to zero — M1
Solve and obtain $t = -2$ — A1
Carry out a complete method for finding the length of $\overrightarrow{PQ}$ — M1
Obtain the given answer $\sqrt{5}$ correctly — A1

OR 1 Calling (4, 2, –1) A, state $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) in component form, e.g. $4\mathbf{i} - 5\mathbf{k}$ — B1
Calculate vector product of $\overrightarrow{AP}$ and a direction vector for l, e.g. $(4\mathbf{i} - 5\mathbf{k}) \times (2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ — M1
Obtain correct answer, e.g. $-5\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$ — A1
Divide modulus of the product by that of the direction vector — M1
Obtain the given answer correctly — A1

OR 2 State $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) in component form — B1
Use a scalar product to find the projection of $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) on l — M1
Obtain correct answer in any form, e.g. $\frac{18}{\sqrt{9}}$ — A1
Use Pythagoras to find the perpendicular — M1
Obtain the given answer correctly — A1

OR 3 State $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) in component form — B1
Use a scalar product to find the cosine of PAQ — M1
Obtain correct answer in any form, e.g. $\frac{18}{\sqrt{41}\cdot\sqrt{9}}$ — A1
Use trig to find the perpendicular — M1
Obtain the given answer correctly — A1

OR 4 State $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) in component form — B1
Find a second point B on l and use the cosine rule in triangle APB to find the cosine of A, B or P, or use a vector product to find the area of APB — M1
Obtain correct answer in any form — A1
Use trig or area formula to find the perpendicular — M1
Obtain the given answer correctly — A1

OR 5 Find $\overrightarrow{PQ}$ for a point Q on l with parameter t, e.g. $4\mathbf{i} - 5\mathbf{k} + t(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ — B1
Use correct method to express PQ^2 (or PQ) in terms of t — M1
Obtain a correct expression in any form, e.g. $(4 + 2t)^2 + (-t)^2 + (-5 - 2t)^2$ — A1
Carry out a complete method for finding its minimum — M1
Obtain the given answer correctly — A1 **5**

10 (i) **EITHER** Use product and chain rule — M1
Obtain correct derivative in any form — A1
OR Square and differentiate LHS by chain rule and RHS by product rule or as powers — M1
Obtain correct result in any form — A1
Set $\frac{dy}{dx}$ equal to zero and make reasonable attempt to solve for $x \neq 0$ — M1
Obtain answer $x = \sqrt{\frac{2}{3}}$, or exact equivalent, correctly — A1 **4**

(ii) State or imply $dx = \cos\theta \, d\theta$ or $\frac{dx}{d\theta} = \cos\theta$ — B1
Substitute for x and dx throughout the integral $\int y dx$ — M1
Obtain the given form correctly with no errors seen — A1 **3**

(iii) Attempt integration and reach indefinite integral of the form $a\theta + b\sin 4\theta$, where $ab \neq 0$ — M1*
Obtain indefinite integral $\frac{1}{8}\theta - \frac{1}{32}\sin 4\theta$, or equivalent — A1
Substitute limits correctly — M1(dep*)
Obtain exact answer $\frac{1}{16}\pi$ — A1 **4**
[Working to carry out the change of limits is needed for the A mark in **(ii)** but, if omitted, can be earned retrospectively if it is seen in part **(iii)**]