UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced/Advanced Subsidiary Level

MARK SCHEME for the May/June 2006 question paper

9709 MATHEMATICS

9709/03 **Paper 3**

Maximum raw mark 75

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

The minimum marks in these components needed for various grades were previously published with these mark schemes, but are now instead included in the Report on the Examination for this session.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2006 question papers for most IGCSE and GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	Use law for the logarithm of a product or quotient, or the logarithm of a power		M1	
	Obtain $\ln x = \ln 4 - y\ln 3$, or equivalent		A1	
	Obtain answer $y = \dfrac{\ln 4 - \ln x}{\ln 3}$, or equivalent		A1	3
2	*EITHER:* State or imply non-modular inequality $(2x)^2 > (x-1)^2$, or corresponding equation		B1	
	Expand and make a reasonable solution attempt at a 2- or 3-term quadratic		M1	
	Obtain critical value $x = \frac{1}{3}$		A1	
	State answer $x > \frac{1}{3}$ only		A1	
	OR: State the relevant critical linear equation, i.e. $2x = 1 - x$		B1	
	Obtain critical value $x = \frac{1}{3}$		B1	
	State answer $x > \frac{1}{3}$		B1	
	State or imply by omission that no other answer exists		B1	
	OR: Obtain the critical value $x = \frac{1}{3}$ from a graphical method, or by inspection, or by solving a linear inequality		B2	
	State answer $x > \frac{1}{3}$		B1	
	State or imply by omission that no other answer exists		B1	4
3	State that $\dfrac{dx}{d\theta} = 2 + 2\cos 2\theta$ or $\dfrac{dy}{d\theta} = 2\sin 2\theta$		B1	
	Use $\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \div \dfrac{dx}{d\theta}$		M1	
	Obtain answer in any correct form, e.g. $\dfrac{2\sin 2\theta}{2 + 2\cos 2\theta}$		A1	
	Make relevant use of sin 2A and cos 2A formulae		M1	
	Obtain given answer correctly		A1	5
4	(i) State answer $R = 25$		B1	
	Use trig formula to find α		M1	
	Obtain $\alpha = 73.74°$		A1	3
	(ii) Carry out evaluation of $\cos^{-1}(15/25)$ $(= 53.1301\ldots°)$		M1	
	Obtain answer 126.9°		A1	
	Carry out correct method for second answer		M1	
	Obtain answer 20.6° and no others in the range		A1√	4
	[Ignore answers outside the given range.]			

5	(i)	State or imply that $\frac{dx}{dt} = kx - 25$	B1	
		Show that $k = 0.1$ and justify the given statement	B1	2
	(ii)	Separate variables and attempt integration	M1	
		Obtain $\ln(x - 250)$, or equivalent	A1	
		Obtain $0.1t$, or equivalent	A1	
		Evaluate a constant or use limits $t = 0$, $x = 1000$ with a solution containing terms $a\ln(x - 250)$ and bt	M1	
		Obtain any correct form of solution, e.g. $\ln(x - 250) = 0.1t + \ln 750$	A1	
		Rearrange and obtain $x = 250(3e^{0.1t} + 1)$, or equivalent	A1	6
6	(i)	Make recognizable sketch of a relevant graph, e.g. $y = 2\cot x$	B1	
		Sketch an appropriate second graph, e.g. $y = 1 + e^{-x}$ correctly and justify the given statement	B1	2
	(ii)	Consider sign of $2\cot x - 1 - e^{-x}$ at $x = 0.5$ and $x = 1$, or equivalent	M1	
		Complete the argument with appropriate calculations	A1	2
	(iii)	Show that the given equation is equivalent to $x = \tan^{-1}\left(\frac{2}{1+e^{-x}}\right)$, or vice versa	B1	1
	(iv)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.61	A1	
		Show sufficient iterations to justify its accuracy to 2d.p., or show there is a sign change in the interval (0.605, 0.615)	A1	3
7	(i)	Show u and u^* in relatively correct positions	B1	
		Show $u + u^*$ in relatively correct position	B1√	
		State or imply that $OACB$ is a parallelogram	B1√	
		State or imply that $OACB$ has a pair of adjacent equal sides	B1√	4
		[The statement that $OACB$ is a rhombus, or equivalent, earns B2√.]		
	(ii)	*EITHER:* Multiply numerator and denominator of $\frac{u}{u^*}$ by $2 + i$	M1	
		Simplify numerator to $3 + 4i$ or denominator to 5	A1√	
		Obtain answer $\frac{3}{5} + \frac{4}{5}i$, or equivalent	A1√	
		OR: Obtain two equations in x and y, and solve for x or for y	M1	
		Obtain $x = \frac{3}{5}$ or $y = \frac{4}{5}$	A1√	
		Obtain answer $\frac{3}{5} + \frac{4}{5}i$	A1√	3
	(iii)	*EITHER:* State or imply $\arg\left(\frac{u}{u^*}\right) = 2\arg u$	M1	
		Justify the given statement correctly	A1	
		OR: Use tan 2A formula with $\tan A = \frac{1}{2}$	M1	
		Justify the given statement correctly	A1	2
		[The fig. is on $-2 + i$ as complex conjugate.]		

8 (i) Use product rule — M1

Obtain derivative in any correct form e.g. $\frac{x^{\frac{1}{2}}}{x}+\frac{x^{-\frac{1}{2}}}{2}.\ln x$ — A1

Equate derivative to zero and solve for $\ln x$ — M1

Obtain $x=e^{-2}$ (or $\frac{1}{e^2}$) or equivalent — A1 — 4

(ii) *EITHER:* Attempt integration by parts with $u=\ln x$ — M1

Obtain $\frac{2}{3}x^{\frac{3}{2}}\ln x-\int\frac{2}{3}x^{\frac{1}{2}}.\frac{1}{x}dx$, or equivalent — A1

OR: Attempt integration by parts with $v=x^{\frac{1}{2}}$ — M1

Obtain $x^{\frac{1}{2}}(x\ln x-x)-\int(x\ln x-x).\frac{x^{-\frac{1}{2}}}{2}dx$ — A1

Obtain indefinite integral $\frac{2}{3}x^{\frac{3}{2}}\ln x-\frac{4}{9}x^{\frac{3}{2}}$, or equivalent — A1

Use $x=1$ and $x=4$ as limits — M1

Obtain answer 4.28 — A1 — 5

9 (i) State or imply partial fractions are of the form $\frac{A}{2-x}+\frac{Bx+C}{1+x^2}$ — B1

Use any relevant method to obtain a constant — M1

Obtain one of the values $A=2$, $B=2$, $C=4$ — A1

Obtain a second value — A1

Obtain the third value — A1 — 5

(ii) Use correct method to obtain the first two terms of the expansion of $(2-x)^{-1}$ or $(1-\frac{1}{2}x)^{-1}$ or $(1+x^2)^{-1}$ — M1

Obtain any correct unsimplified expansion of the partial fractions up to the terms in x^3, e.g. $(2x+4)(1+(-1)x^2)$ (deduct A1 for each incorrect expansion) — A1√ + A1√

Carry out multiplication of expansion of $(1+x^2)^{-1}$ by $(2x+4)$ — M1

Obtain answer $5+\frac{5}{2}x-\frac{15}{4}x^2-\frac{15}{8}x^3$ — A1 — 5

[Binomial coefficients involving -1, e.g. $\binom{-1}{1}$, are not sufficient for the M1 mark. The f.t. is on A, B, C.]

[In the case of an attempt to expand $10(2-x)^{-1}(1+x^2)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]

[Allow the use of Maclaurin, giving M1A1√ for $f(0)=5$ and $f'(0)=\frac{5}{2}$, A1√ for $f''(0)=-\frac{15}{2}$, A1√ for $f'''(0)=-\frac{45}{4}$, and A1 for obtaining the correct final answer (f.t. is on A, B, C if used).]

10	(i)	State $\mathbf{r} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix}$, or equivalent	B1	1
	(ii)	Express $\overrightarrow{BN}$ in terms of λ, e.g. $\begin{pmatrix} -1+3\lambda \\ 3-\lambda \\ 5-4\lambda \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ -4 \end{pmatrix}$, or equivalent	B1	
		Equate its scalar product with $\begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix}$ to zero and solve for λ	M1	
		Obtain $\lambda = 2$	A1	
		Obtain $\overrightarrow{ON} = \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix}$, or equivalent	A1√	
		Carry out method for calculating BN, i.e. $\lvert 2\mathbf{i} - 2\mathbf{j} + \mathbf{k} \rvert$	M1	
		Obtain the given answer $BN = 3$ correctly	A1	6
	(iii)	*EITHER:* Use scalar product to obtain a relevant equation in a, b and c, e.g. $3a - b - 4c = 0$ or $2a + 2b + c = 0$	M1	
		State two correct equations in a, b, c	A1√	
		Solve simultaneous equations to obtain one ratio, e.g. $a : b$	M1	
		Obtain $a : b : c = 7 : -11 : 8$, or equivalent	A1	
		Obtain equation $7x - 11y + 8z = 0$, or equivalent	A1	
		OR: Substitute for A, B and N in equation of plane and state 3 equations in a, b, c, d	B1	
		Eliminate one unknown, e.g. d, entirely and obtain 2 equations in 3 unknowns	M1	
		Solve to obtain one ratio e.g. $a : b$	M1	
		Obtain $a : b : c = 7 : -11 : 8$, or equivalent	A1	
		Obtain equation $7x - 11y + 8z = 0$, or equivalent	A1	
		OR: Calculate vector product of two vectors parallel to the plane, e.g. $(3\mathbf{i} - \mathbf{j} - 4\mathbf{k}) \times (2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	M1	
		Obtain 2 correct components of the product	A1√	
		Obtain correct product e.g. $7\mathbf{i} - 11\mathbf{j} + 8\mathbf{k}$, or equivalent	A1	
		Substitute in $7x - 11y + 8z = d$ and find d, or equivalent	M1	
		Obtain equation $7x - 11y + 8z = 0$, or equivalent	A1	
		OR: Form correctly a 2-parameter equation for the plane	M1	
		Obtain equation in any correct form e.g. $\mathbf{r} = -\mathbf{i} + 3\mathbf{j} + 5\mathbf{k} + \lambda(3\mathbf{i} - \mathbf{j} - 4\mathbf{k}) + \mu(2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	A1√	
		State 3 equations in x, y, z, λ and μ	A1	
		Eliminate λ and μ	M1	
		Obtain equation $7x - 11y + 8z = 0$, or equivalent	A1	5