**HIGHER MATHEMATICS**     **8719/07**

**MATHEMATICS**     **9709/07**

Paper 7  Probability & Statistics 2  **(S2)**

May/June 2004

**1 hour 15 minutes**

Additional materials:  Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.

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1 Each multiple choice question in a test has 4 suggested answers, exactly one of which is correct. Rehka knows nothing about the subject of the test, but claims that she has a special method for answering the questions that is better than just guessing. There are 60 questions in the test, and Rehka gets 22 correct.

   **(i)** State null and alternative hypotheses for a test of Rehka's claim. [1]

   **(ii)** Using a normal approximation, test at the 5% significance level whether Rehka's claim is justified. [4]

2 In athletics matches the triple jump event consists of a hop, followed by a step, followed by a jump. The lengths covered by Albert in each part are independent normal variables with means 3.5 m, 2.9 m, 3.1 m and standard deviations 0.3 m, 0.25 m, 0.35 m respectively. The length of the triple jump is the sum of the three parts.

   **(i)** Find the mean and standard deviation of the length of Albert's triple jumps. [3]

   **(ii)** Find the probability that the mean of Albert's next four triple jumps is greater than 9 m. [3]

3 The independent random variables $X$ and $Y$ are such that $X$ has mean 8 and variance 4.8 and $Y$ has a Poisson distribution with mean 6. Find

   **(i)** $E(2X - 3Y)$, [2]

   **(ii)** $Var(2X - 3Y)$. [4]

4 Packets of cat food are filled by a machine.

   **(i)** In a random sample of 10 packets, the weights, in grams, of the packets were as follows.

   374.6   377.4   376.1   379.2   371.2   375.0   372.4   378.6   377.1   371.5

   Find unbiased estimates of the population mean and variance. [3]

   **(ii)** In a random sample of 200 packets, 38 were found to be underweight. Calculate a 96% confidence interval for the population proportion of underweight packets. [4]

5 The lectures in a mathematics department are scheduled to last 54 minutes, and the times of individual lectures may be assumed to have a normal distribution with mean $\mu$ minutes and standard deviation 3.1 minutes. One of the students commented that, on average, the lectures seemed too short. To investigate this, the times for a random sample of 10 lectures were used to test the null hypothesis $\mu = 54$ against the alternative hypothesis $\mu < 54$ at the 10% significance level.

   **(i)** Show that the null hypothesis is rejected in favour of the alternative hypothesis if $\bar{x} < 52.74$, where $\bar{x}$ minutes is the sample mean. [4]

   **(ii)** Find the probability of a Type II error given that the actual mean length of lectures is 51.5 minutes. [4]

**6** At a certain airfield planes land at random times at a constant average rate of one every 10 minutes.

   **(i)** Find the probability that exactly 5 planes will land in a period of one hour. [2]

  **(ii)** Find the probability that at least 2 planes will land in a period of 16 minutes. [3]

 **(iii)** Given that 5 planes landed in an hour, calculate the conditional probability that 1 plane landed in the first half hour and 4 in the second half hour. [3]

**7** The queuing time, $T$ minutes, for a person queuing at a supermarket checkout has probability density function given by

$$f(t) = \begin{cases} ct(25 - t^2) & 0 \leqslant t \leqslant 5, \\ 0 & \text{otherwise,} \end{cases}$$

where $c$ is a constant.

  **(i)** Show that the value of $c$ is $\frac{4}{625}$. [3]

  **(ii)** Find the probability that a person will have to queue for between 2 and 4 minutes. [3]

 **(iii)** Find the mean queuing time. [4]

**4**

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