

General Certificate of Education
June 2008
Advanced Level Examination



STATISTICS
Unit Statistics 5

SS05

Monday 16 June 2008 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is SS05.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 A nurse measures the heights of schoolchildren to the nearest centimetre. Rounding error and a slight bias in the measuring technique cause errors in the recorded heights. These errors, in millimetres, may be modelled by the random variable X , with probability density function

$$f(x) = \begin{cases} 0.1 & -2 < x < 8 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the probability that an error lies between 4 mm and 8 mm. *(2 marks)*
- (b) Find the probability that the magnitude of an error (ie ignore the sign) is less than 4 mm. *(2 marks)*
- (c) Find the mean and the standard deviation of X . *(3 marks)*
- 2 Akiva bought a set of digital bathroom scales which measure weight to the nearest gram. To test the scales' reliability, Akiva repeatedly weighed himself before taking a bath. The results of his seven weighings, in grams, were

84 691 84 735 84 796 84 742 84 764 84 709 84 751

- (a) Regarding these results as a random sample from a normal distribution, construct a 95% confidence interval for:
- (i) the standard deviation; *(7 marks)*
- (ii) the mean, giving your answer to an appropriate level of accuracy. *(6 marks)*
- (b) When buying the scales, Akiva had been offered a more expensive set which were guaranteed to give a standard deviation of not more than 15 grams for repeated weighings of the same person.

Using your confidence interval in part (a)(i), compare the variability of the two sets of scales when recording repeated weighings of the same person. Comment on whether you think it would have been advisable for Akiva to buy the more expensive scales.

(3 marks)

- (c) Akiva was surprised that the recorded weighings were so high. He wondered whether the scales were biased. What information, if any, can be deduced from your confidence interval in part (a)(ii) about whether or not the scales were biased? Explain your answer. *(1 mark)*

- 3 Investigation of a number of accidents that had occurred on a fairly quiet stretch of country road suggested that excessive speed had been an important factor. Consequently, a speed camera was installed on this stretch of road. The speeds, in mph, of random samples of vehicles before and after the installation of the speed camera were recorded.

Before speed camera installed	61.4	70.5	78.2	58.5	66.3	84.6	69.4
After speed camera installed	47.2	59.4	62.5	38.3	68.6	58.4	

- (a) By carrying out a suitable test at the 5% significance level, verify that it is reasonable to assume that there is no difference between the standard deviations of the speeds before and after the installation of the speed camera. Assume that both samples come from normal distributions. *(8 marks)*
- (b) Test, at the 5% significance level, whether the installation of the speed camera has reduced the average speed of vehicles on this stretch of road. *(9 marks)*
- (c) When the sample was taken after the speed camera had been installed, the speed of one vehicle was recorded as 11.3 mph. This vehicle had slowed down to avoid sheep that had wandered on to the road. It was decided to exclude this speed from the sample.

Comment on this decision. **No further calculations are required.** *(2 marks)*

- 4 The time to failure, in hours, of a drill bit used in tunnelling machinery may be modelled by an exponential distribution with parameter $\lambda = 0.02$. Drill bits are used continuously until they fail.

- (a) Find the mean time to failure of a drill bit. *(2 marks)*
- (b) Find the probability that a drill bit will fail during an eight-hour shift. *(3 marks)*
- (c) Find the probability of a drill bit **not** failing during 5 consecutive eight-hour shifts. *(3 marks)*
- (d) It is suggested that the risk of a drill bit failing during a shift could be reduced by always using a new one at the start of each shift. Comment on this suggestion. *(2 marks)*
- (e) The number of drill bits which fail during 5 consecutive eight-hour shifts may be modelled by a Poisson distribution.
- (i) Find the mean of this Poisson distribution. *(2 marks)*
- (ii) Hence find the probability of no drill bits failing during 5 consecutive eight-hour shifts. *(1 mark)*

Turn over for the next question

- 5 Students applying for part-time jobs at an insurance company are given an aptitude test consisting of 8 simple tasks. Ebony is in charge of recruitment to part-time jobs and, for each applicant, she decides whether or not each task has been carried out satisfactorily. The numbers of tasks judged by Ebony to have been carried out unsatisfactorily by a random sample of 120 applicants are summarised in the table.

Number of tasks carried out unsatisfactorily	0	1	2	3	4	5	6	7	8
Number of applicants	5	21	56	10	19	5	4	0	0

- (a) Show that 0.30 is a suitable estimate of the proportion of tasks judged by Ebony to be carried out unsatisfactorily. *(2 marks)*
- (b) Using a χ^2 distribution and the 1% significance level, verify that the binomial distribution does **not** provide a suitable model for the number of tasks judged by Ebony to have been carried out unsatisfactorily by an applicant. *(12 marks)*
- (c) Does your result support the theory that the probability of an applicant failing to carry out a task satisfactorily is constant for all applicants and all tasks? Explain your answer. *(2 marks)*
- (d) Any applicant who is judged to be unsatisfactory on 3 or more tasks is not considered for employment. Examine the differences between your observed and expected values and hence suggest a possible reason why the binomial distribution does not provide a suitable model for the data. *(3 marks)*

END OF QUESTIONS