General Certificate of Education
June 2006
Advanced Level Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

STATISTICS **SS05**
Unit Statistics 5

Wednesday 21 June 2006 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is SS05.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A factory produces hand-made chocolate truffles. The weights, in grams, of a random sample of 12 truffles are:

19.3 19.4 19.7 18.6 20.5 19.2 19.6 18.5 18.8 18.9 18.5 20.0

(a) Calculate an unbiased estimate, s^2, of the variance of the weights of chocolate truffles. *(2 marks)*

(b) Stating the necessary distributional assumption, calculate a 95% confidence interval for the variance of the weights of chocolate truffles. *(6 marks)*

(c) Hence comment on the statement that the standard deviation of the weights of chocolate truffles is 0.4 grams. *(2 marks)*

2 The Number 6 bus and the Number 23 bus both travel between the town centre and the hospital, but they follow different routes. For each route, the journey time from the town centre to the hospital is a normally distributed random variable with standard deviation 1.8 minutes.

Melanie travels regularly from the town centre to the hospital, taking whichever bus goes first. She records her journey times for 15 journeys on a Number 6 bus and 10 journeys on a Number 23 bus. She then calculates the mean journey time for each route, with the following results.

Number 6 bus:	Sample size = 15	Sample mean = 13.2 minutes
Number 23 bus:	Sample size = 10	Sample mean = 14.6 minutes

(a) Use a test, at the 5% level of significance, to determine whether the mean time taken to travel from the town centre to the hospital is the same for both bus routes. *(7 marks)*

(b) State **one** assumption that must be made about Melanie's samples of data for the hypothesis test in part (a) to be valid. *(1 mark)*

(c) State, in the context of this question, the meaning of a Type II error. *(1 mark)*

3 In a survey of motorway traffic, the speeds of cars are recorded correct to the nearest 5 kilometres per hour (kph). The exact speed of a car whose speed is recorded as 100 kph is denoted by X kph. The random variable X may be modelled by a rectangular distribution on the interval $[97.5, 102.5]$.

(a) (i) State the probability density function of X. *(1 mark)*

(ii) Find the standard deviation of X. *(2 marks)*

(b) A car is chosen at random from those with speeds recorded as 100 kph. Find the probability that its exact speed is between 98 kph and 101 kph. *(2 marks)*

(c) For a random sample of 50 cars, each travelling at a speed recorded as 100 kph, the mean speed is denoted by $\overline{X}$ kph.

Specify fully a distribution which may be used to model the variable $\overline{X}$, giving a reason for your choice of distribution. *(3 marks)*

4 The numbers of calls per hour made to a telephone helpline are recorded. The results for a random sample of 80 one-hour periods during night shifts are shown in the table.

Number of calls	0	1	2	3	4	5	6
Frequency	1	9	23	19	17	6	5

(a) Use a χ^2 goodness of fit test, at the 10% level of significance, to show that the numbers of calls per hour during night shifts may be modelled by a Poisson distribution with mean 3. *(12 marks)*

(b) Gisela is given the above frequency table and calculates the sample mean to be exactly 3 calls per hour. She then carries out a χ^2 goodness of fit test, at the 10% level of significance, to investigate whether the data can be modelled by a Poisson distribution.

(i) Explain why the critical value of χ^2 for Gisela's test will be different from the one that you used in part (a). *(2 marks)*

(ii) State the conclusion that Gisela should reach. *(1 mark)*

(c) A night shift lasts for 8 hours and calls are equally likely to occur at any time during the night.

Find an estimate of the probability that more than 30 calls are received during a particular night shift. *(4 marks)*

Turn over for the next question

5 The lifetime of a HiPower light is T **thousand** hours, where T has an exponential distribution with parameter $\lambda = 0.8$.

(a) Find:

(i) the mean value of T; *(2 marks)*

(ii) the mean lifetime of a HiPower light; *(1 mark)*

(iii) the probability that a HiPower light has a lifetime of less than 1000 hours. *(3 marks)*

(b) HiPower lights are used throughout a large office building.

(i) A light has been in use for 500 hours. Find the probability that it lasts for at least another 500 hours. *(3 marks)*

(ii) Three new lights are installed in one of the offices. Find the probability that at least one of them fails within 500 hours. *(3 marks)*

6 A fast-food chain sells ChoiceBurgers and SlimBurgers. The fat contents, X grams per ChoiceBurger and Y grams per SlimBurger, are normally distributed random variables.

Stephen, a nutritionist, examines a random sample of 11 ChoiceBurgers and records the fat content, x grams, for each burger. He also examines a random sample of 9 SlimBurgers and records the fat content, y grams, for each burger. From these results he obtains the following information.

ChoiceBurgers:	$\bar{x} = 36.8$	and	$s_x^2 = 3.798$
SlimBurgers:	$\bar{y} = 29.1$	and	$s_y^2 = 2.925$

(a) Carry out a test, at the 10% level of significance, to show that it is reasonable to believe that the variances of X and Y are equal. *(6 marks)*

(b) The fast-food chain states that the mean fat content per SlimBurger is 10 grams less than the mean fat content per ChoiceBurger. Stephen claims that this difference in mean fat content is smaller than 10 grams.

Assuming that X and Y have equal variances, apply a test, at the 1% level of significance, to investigate Stephen's claim. *(11 marks)*

END OF QUESTIONS