Version 1.0: 0606

ASSESSMENT and QUALIFICATIONS ALLIANCE

# General Certificate of Education

# Statistics 6380

*SS05 Statistics 5*

# Mark Scheme

*2006 examination - June series*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

## Key To Mark Scheme And Abbreviations Used In Marking

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |

| | | | |
|---|---|---|---|
| √or ft or F | follow through from previous incorrect result | MC | mis-copy |
| CAO | correct answer only | MR | mis-read |
| CSO | correct solution only | RA | required accuracy |
| AWFW | anything which falls within | FW | further work |
| AWRT | anything which rounds to | ISW | ignore subsequent work |
| ACF | any correct form | FIW | from incorrect work |
| AG | answer given | BOD | given benefit of doubt |
| SC | special case | WR | work replaced by candidate |
| OE | or equivalent | FB | formulae book |
| A2,1 | 2 or 1 (or 0) accuracy marks | NOS | not on scheme |
| $-x$ EE | deduct $x$ marks for each error | G | graph |
| NMS | no method shown | c | candidate |
| PI | possibly implied | sf | significant figure(s) |
| SCA | substantially correct approach | dp | decimal place(s) |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

## SS05

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **1(a)** | $s^2 = 0.3955$ | B2 | 2 | AWFW 0.395 to 0.396 |
| **(b)** | Assuming the weights of truffles are normally distributed: | B1 | | Must indicate that population is normally distributed |
| | $\nu = 12 - 1 = 11$ | B1 | | |
| | CVs for $\chi^2$ are 3.816, 21.920 | B1 | | Both; accept 3.82, 21.9 |
| | confidence limits for $\sigma^2$ are | | | |
| | $\frac{11 \times 0.3955}{21.920}, \frac{11 \times 0.3955}{3.816}$ | M1<br>A1√ | | √ on $s^2$ and $\chi^2$ values |
| | giving confidence interval (0.198, 1.14) | A1 | 6 | (0.198 to 0.199, AWRT) |
| **(c)** | sd of 0.4 $\Rightarrow$ variance of 0.16 which is below CI so statement is unlikely to be true. | E1√<br>B1√ | 2 | √ on CI<br>√ on CI<br>B1 if CI for standard deviation found but incorrectly used. |
| | **Total** | | **10** | |
| **2(a)** | $X$ = time in minutes for bus 6 journey<br>$Y$ = time in minutes for bus 23 journey | | | |
| | $H_0$: $\mu_X = \mu_Y$ | B1 | | Compares population means |
| | $H_1$: $\mu_X \neq \mu_Y$ | B1 | | = and ≠ |
| | CVs of $z$ are ±1.96 | B1 | | 1.96 accepted as implying 2-tail |
| | test statistic = $\frac{13.2 - 14.6}{\sqrt{\frac{1.8^2}{15} + \frac{1.8^2}{10}}}$ | M1<br>A1 | | Subtraction either way if used consistently |
| | $= -1.905$ | A1 | | AWFW –1.91 to –1.90 |
| | Not enough evidence to suggest a difference in mean journey times. | A1√ | 7 | √ on CV and test statistic<br>Only if 2-tail test used |
| **(b)** | Random samples from population of journey times or samples are independent. | B1 | 1 | Accept valid comment in context implying samples are representative |
| **(c)** | Concluding there is no difference in mean journey times when there is a difference. | B1 | 1 | |
| | **Total** | | **9** | |

**SS05 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **3(a)(i)** | $f(x)=\begin{cases}0.2 & 97.5\le x\le 102.5\\ 0 & \text{otherwise}\end{cases}$ | B1 | 1 | Mark given for finding 0.2 without formal expression |
| **(ii)** | $\text{sd}=\sqrt{\dfrac{(102.5-97.5)^2}{12}}$ | M1 | | |
| | $=\dfrac{5}{\sqrt{12}}$ =1.44 | A1 | 2 | AWRT |
| **(b)** | $P(98\le X\le 101)=\dfrac{101-98}{102.5-97.5}$ | M1 | | |
| | $=\dfrac{3}{5}=0.6$ | A1 | 2 | CAO |
| **(c)** | $\bar{X}\sim N\left(100,\dfrac{25}{12\times 50}\right)$ | B1<br>B1 | | Normal distribution<br>Mean = 100; Variance from (a)(ii) divided by 50 |
| | $=N\left(100,\dfrac{1}{24}\right)$ | | | |
| | Large sample so central limit theorem applies. | E1 | 3 | Mark given for mean of large sample or reference to CLT |
| | **Total** | | **8** | |

**SS05 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **4(a)** | $H_0$: Number of calls, $X$, ~ Poisson(3)<br>$H_1$: not $H_0$ | B1 | | Must specify which Poisson<br><br>In table: accept probabilities by formula or tables to at least 3 dp; $E$ values to at least 2 dp. |

| $x$ | $O$ | $P(X=x)$ | $E$ | $\frac{(O-E)^2}{E}$ | Marks | Comments |
|---|---|---|---|---|---|---|
| 0 | 1 } | 0.0498 | 3.984 } | 2.206 | M1 | Poisson probabilities |
| 1 | 9 } | 0.1493 | 11.944 } | | M1 | $E$ = Probability × 80 |
| 2 | 23 | 0.2241 | 17.928 | 1.435 | M1 | Combining first two groups |
| 3 | 19 | 0.2240 | 17.920 | 0.065 | | ($O$ =10; $E$ = 15.92 to 15.93) |
| 4 | 17 | 0.1681 | 13.448 | 0.938 | m1 | Last group ≥ 6; dependent on first M1 |
| 5 | 6 | 0.1008 | 8.064 | 0.528 | A2 | 7 $E$s AWRT 1 dp (A1 if 5 or 6); lose one A mark for too much rounding |
| ≥ 6 | 5 | 0.0839 | 6.712 | 0.437 | | |
| | 80 | 1 | 80 | 5.609 | | |

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| | $\chi^2 = \sum \frac{(O-E)^2}{E} = 5.609$ | M1<br>A1√ | | Use of formula<br>AWFW 5.60 to 5.71; √ only on minor calculation error |
| | DF $\nu = (7-1)-1 = 5$ | B1 | | No. of categories used –1 |
| | CV of $\chi^2 = 9.236$ | B1√ | | √ on $\nu$ |
| | Insufficient evidence at 10% level to say that Poisson (3) is not a suitable model. | A1√ | 12 | √ on CV and tabulated total<br>Accept Poisson with no parameter |
| **(b)(i)** | Same number of categories but one more constraint so $\nu = 4$, giving | E1 | | 1 less than DF in (a) |
| | $\chi^2 = 7.779$ | B1 | 2 | $\chi^2$ matching $\nu$, seen here or in (ii) |
| **(ii)** | Total from table unchanged and is < 7.779 so conclusion is same as in (a) | B1√ | 1 | √ on values from (a) if consistent; conclusion must be justified |
| **(c)** | Number of calls, $Y$, in 8 hours ~ Poisson(24) ≈ N(24, 24) | B1 | | |
| | $P(Y > 30) = 1 - \Phi\left(\frac{30.5-24}{\sqrt{24}}\right)$ | M1<br>A1 | | Continuity correction |
| | $= 1 - \Phi(1.33)$<br>$= 0.092$ | A1√ | 4 | √ on lack of c. c. only<br>SC B1 if try to use normal approximation, but wrong variance |
| | **Alternatively**<br>By calculator from Poisson:<br>$P(Y > 30) = 1 - P(Y \le 30)$<br>$= 1 - 0.90415 = 0.0959$ | (B4) | | AWRT 0.096 |
| | **Total** | | **19** | |

**SS05 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **5(a)(i)** | $\mathrm{E}(T) = \dfrac{1}{0.8}$ | M1 | | |
| | $= 1.25$ | A1 | 2 | |
| **(ii)** | Mean lifetime = | | | |
| | $1.25 \times 1000 = 1250$ hours | A1√ | 1 | √ on E($T$) |
| | | | | Condone lack of units |
| **(iii)** | P(lifetime < 1000) = P($T$ < 1) | B1 | | Using $t$ = 1 or mean = 1250 |
| | $\mathrm{P}(T \le t) = 1 - \mathrm{e}^{-0.8t}$ | | | Or $\lambda = 0.0008$ |
| | $\mathrm{P}(T \le 1) = 1 - \mathrm{e}^{-0.8}$ | M1 | | Attempt to use F($x$) or F($t$) or integration |
| | $= 0.551$ | A1 | 3 | 0.550 to 0.551 |
| **(b)(i)** | Exponential distribution has 'no memory' so required probability is | | | |
| | P($T$ > 0.5) | M1 | | Use of this property; P($T$ > 1) or $\mathrm{P}(X > 500)$ seen |
| | $= 1 - \left(1 - \mathrm{e}^{-0.4}\right)$ | m1 | | 1–F $(0.5)$ or $\mathrm{F}(500)$ attempted |
| | $= 0.670$ | A1 | 3 | 0.670 to 0.671 |
| **(ii)** | P(at least one fails within 500 hours) | | | |
| | = 1 – P(all last more than 500 hours) | M1 | | Or combining probs. For 1,2 and 3 failures |
| | $= 1 - \left(\mathrm{e}^{-0.4}\right)^3$ | | | |
| | $= 1 - (0.670)^3$ | m1 | | Or use of multiplication and addition laws |
| | $= 0.699$ | A1√ | 3 | AWRT 0.70; √ on sensible probability |
| | **Total** | | **12** | |

**SS05 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **6(a)** | $H_0$: $\sigma_X{}^2 = \sigma_Y{}^2$<br>$H_1$: $\sigma_X{}^2 \neq \sigma_Y{}^2$ | B1 | | Or $H_0$: $\sigma_X = \sigma_Y$<br>$H_1$: $\sigma_X \neq \sigma_Y$; (both) |
| | DF: $\nu_1 = 10$, $\nu_2 = 8$ | B1 | | |
| | CV of $F$ is 3.347 | B1 | | Both B1 if dof reversed with matching CV |
| | $\frac{s_X{}^2}{s_Y{}^2} = \frac{3.798}{2.925}$ | M1 | | |
| | $= 1.30$ | A1 | | AWRT |
| | < CV so reasonable to believe that $\sigma_X{}^2 = \sigma_Y{}^2$ | E1 | 6 | Justifies conclusion; accept appropriate diagram |
| **(b)** | $H_0$: $\mu_X - \mu_Y = 10$<br>$H_1$: $\mu_X - \mu_Y < 10$ | B2,1 | | B1 For saying difference of means = 10; <10 without saying which way subtracted or for $X$ and $Y$ reversed |
| | Pooled estimate of variance $= \frac{(10 \times 3.798) + (8 \times 2.925)}{18}$ | M1 | | |
| | = 3.41 | A1 | | |
| | DF $\nu = 18$ | B1 | | |
| | CV of $t = -2.552$ | B1 | | Accept + 2.552 here |
| | test statistic = $\frac{(36.8 - 29.1) - 10}{\sqrt{3.41\left(\frac{1}{11} + \frac{1}{9}\right)}}$ | M1<br>A1<br>A1√ | | Use of correct formula<br>For –10<br>Correct values substituted; √ on pooled estimate |
| | $= -2.77$ | A1√ | | √ on pooled estimate |
| | $< -2.552$ so supports Stephen's claim that the difference in mean fat contents is less than 10 grams. | A1√ | 11 | √ on test statistic and CV |
| | **Total** | | **17** | |
| | **TOTAL** | | **75** | |