General Certificate of Education (A-level) June 2012

## Statistics

SS04

## (Specification 6380)

Statistics 4

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

[^0]Copyright © 2012 AQA and its licensors. All rights reserved.

## Copyright

AQA retains the copyright on all its publications. However, registered schools/colleges for AQA are permitted to copy material from this booklet for their own internal use, with the following important exception: AQA cannot give permission to schools/colleges to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Set and published by the Assessment and Qualifications Alliance.

## Key to mark scheme abbreviations

| M | mark is for method |
| :--- | :--- |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| Jor ft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0) accuracy marks |
| $-x$ EE | deduct $x$ marks for each error |
| NMS | no method shown |
| PI | possibly implied <br> SCA |
| substantially correct approach |  |
| cf | candidate |
| dp | significant figure(s) |
| decimal place(s) |  |

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award full marks. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn no marks.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns full marks, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains no marks.

Otherwise we require evidence of a correct method for any marks to be awarded.

## SS04

| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Approximate 95\% confidence interval | B1 |  | Use of $\sqrt{ } 86$ for s.d. |
|  | $\begin{aligned} & 86 \pm 1.96 \sqrt{ } 86 \\ & 86 \pm 18.18 \end{aligned}$ | M1 <br> B1 |  | Method for confidence interval, their s.d. $1.96$ |
|  | $67.8 \sim 104.2$ | A1 | 4 | 67.8 ( $67.8 \sim 68$ ) and 104.2 ( 104 ~ 104.2) Allow in $\pm$ form |
|  | Total |  | 4 |  |


| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 2 (a) | $\bar{x}=28.791 \quad s=2.4060$ | B1 |  | $\begin{aligned} & \hline 28.79(28.75 \sim 28.85) \\ & \text { and } 2.406(2.4 \sim 2.41) \end{aligned}$ |
|  | $\mathrm{H}_{0}: \mu=30.5 \mathrm{H}_{1}: \mu \neq 30.5$ | B1 |  | Both hypotheses |
|  | $t=\frac{(28.791-30.5)}{2.4060 / \sqrt{11}}$ | $\begin{aligned} & \text { M1 } \\ & \text { m1 } \end{aligned}$ |  | Use of their s.d. $/ \sqrt{ } 11$ <br> Method for $t$ - ignore sign |
|  | $=-2.36$ | A1 |  | -2.36 (-2.35 ~-2.36) |
|  | c.v. $t_{10} \pm 2.228$ | $\begin{aligned} & \text { B1 } \\ & \text { B1 } \checkmark \end{aligned}$ |  | 10df <br> $\pm 2.228$ their df - ignore sign |
|  | Reject $\mathrm{H}_{0}$ There is significant evidence that the mean length of male student's feet is less than (or not equal) 30.5 cm . | A1 $\checkmark$ | 8 | Conclusion must be compared with lower tail of $t$ and not inconsistent with their $\mathrm{H}_{0}$. <br> Allow arithmetic errors and incorrect $t$-values only |
| (b) | Assuming the sample is representative, there is evidence that the mean length of male student's feet is less than/not a | E1 |  | Mean less than/not a foot - may be earned in (a) |
|  | some male students' feet will be greater than a foot (and some equal to a foot to some reasonable level of | E1 | 2 | Some individuals greater or equal to a foot |
|  | accuracy). |  |  | Allow a mark for other sensible comments eg only left feet measured - max 2 |
|  | Total |  | 10 |  |


| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 3(a) | $p=22 / 125=0.176$ | B1 |  | 22/125 acf |
|  | 95\% confidence interval for $p$ | M1 |  | Method for s.d. |
|  | $0.176 \pm 1.96 \sqrt{\frac{0.176 \times 0.824}{125}}$ | $\begin{aligned} & \mathrm{B} 1 \\ & \mathrm{~m} 1 \end{aligned}$ |  | $\begin{aligned} & 1.96 \\ & \text { Method - allow incorrect } z \text {-value } \end{aligned}$ |
|  | $0.176 \pm 0.0668$ |  |  |  |
|  | $0.109 \sim 0.243$ | A1 | 5 | $\begin{aligned} & 0.109(0.109 \sim 0.11) \\ & \text { and } 0.243(0.242 \sim 0.243) \\ & \text { Allow in } \pm \text { form } \end{aligned}$ |
| (b) | $p=91 / 125=0.728$ |  |  |  |
|  | 95\% confidence interval for $p$ |  |  |  |
|  | $0.728 \pm 1.96 \sqrt{\frac{0.728 \times 0.272}{125}}$ | M1 |  | Method - allow incorrect z- value |
|  | $0.728 \pm 0.0780$ |  |  |  |
|  | $0.650 \sim 0.806$ | A1 | 2 | $\begin{aligned} & 0.650(0.645 \sim 0.655) \\ & \text { and } 0.806(0.805 \sim 0.807) \\ & \text { Allow in } \pm \text { form } \end{aligned}$ |
| (c)(i) | No significant evidence of difference between USA and England for first statement since 0.21 lies between 0.109 and 0.243 | B1 E1 $\checkmark$ |  | No difference In interval |
| (ii) | No significant evidence of difference between USA and England for second statement since 0.66 lies between 0.650 and 0.806 | B1 |  | No difference |
| (iii) | Significant evidence that more agreed with second statement than first since confidence interval for agreement with second statement is wholly above that for first statement. | B1 E1 | 5 | Evidence of difference No overlap of c.i. |
|  | Total |  | 12 |  |


| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 4(a)(i) | Binomial $n=400 p=0.0075$ <br> $\rightarrow$ Poisson, mean $400 \times 0.0075=3$ | $\begin{gathered} \text { M1 } \\ \text { B1 } \end{gathered}$ |  | Attempt at Poisson approx. mean 3 |
|  | $\mathrm{P}(>4)=1-0.8153=0.185$ | A1 | 3 | 0.185 (0.18~19) |
| (ii) | Binomial $n=400 \quad p=0.75$ <br> $\rightarrow$ Normal, mean $400 \times 0.75=300$ <br> s.d. $=\sqrt{ }(400 \times 0.75 \times 0.25)=8.660$ | $\begin{aligned} & \text { M1 } \\ & \text { m1 } \end{aligned}$ |  | Attempt at normal approx. <br> Method for mean and s.d./variance |
|  | $\begin{aligned} & z_{1}=(289.5-300) / 8.660=-1.212 \\ & z_{2}=(305.5-300) / 8.660=0.635 \end{aligned}$ | B1 |  | Attempt at continuity correction |
|  |  | M1 |  | Method - ignore omitted or incorrect cc |
|  | $\begin{aligned} & \mathrm{P}(\geq 290 \text { and } \leq 305) \\ &=0.7373-(1-0.8874) \end{aligned}$ | m1 |  | Completely correct method |
|  | $=0.625$ | A1 | 6 | 0.625 (0.62 ~ 0.63 ) |
| (b) | both approximations give answers close to the exact answers. | $\begin{gathered} \text { E1 } \sqrt{\text { E1 }} \end{gathered}$ | 2 | Sensible comment - their answers Both good approximations |
| (c)(i) | Approximate confidence interval for proportions | E1 | 1 | c.i. for proportions |
| (ii) | There is no practical use for Poisson approximation to binomial if you possess a calculator which will calculate all binomial probabilities exactly | E2(1) | 2 | Both marks for well expressed answer. Also allow credit for needed for exam/helps understanding of relationship between binomial and Poisson etc |
|  | Total |  | 14 |  |


| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 5(a)(i) | $z=(500-509) / 4.5=-2.0$ <br> probability bottle contains less than 500 grams of beer is $1-0.97725$ | M1 |  | Method - allow wrong tail |
|  | $=0.02275$ | A1 |  | 0.02275 (0.0225 ~ 0.023) |
| (ii) | $\mathrm{P}($ not exactly 500$)=1$ | B1 |  | cao |
| (iii) | distribution of $X+Y$ is normal mean $509+446=955$ | B1 |  | 955 cao |
|  | s.d. $\sqrt{ }\left(4.5^{2}+5.6^{2}\right)=7.184$ (variance 51.61) $z=(950-955) / 7.184=-0.696$ | M1 m1 |  | Method for s.d./variance Method - allow wrong tail |
|  | probability bottle + beer weighs more than 950 grams $=0.757$ | A1 | 7 | 0.757 ( $0.754 \sim 0.76)$ |
| (b)(i) | normal, mean 955-446=509 | B1 |  | 509 cao |
|  | s.d. $\sqrt{ }\left(5.6^{2}+6.2^{2}\right)=8.355$ <br> (variance 69.8) | M1 | 2 | Method for s.d./variance |
| (ii) | $z=\frac{(500-509)}{8.355}=-1.077$ | m1 |  | Method allow wrong tail |
|  | probability bottle contains less than 500 grams of beer is $1-0.8594$ $=0.141$ | A1 | 2 | 0.141 ( 0.14~0.143) |
| (c) | Amount of beer in bottle is more variable with new machine. | E2(1) | 2 | Amount of beer more variable. Allow a comparison of probabilities in (a)(i) and (b)(ii) |
|  | Total |  | 13 |  |


| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 6(a) | $\mathrm{H}_{0}: p=0.45 \mathrm{H}_{1}: p>0.45$ | B1 |  | Hypotheses |
|  | $\mathrm{B}(15,0.45)$ | M1 |  | Use of $\mathrm{B}(15,0.45)$ |
|  | probability $\geq 9=1-0.8182=0.1818$ | A1 |  | 0.181 (0.181~0.182) |
|  | $0.1818>0.05$ accept $\mathrm{H}_{0}$ : no significant evidence that Easter classes have improved pass rate | $\begin{aligned} & \text { A1 } \checkmark \\ & \text { A1 } \checkmark \end{aligned}$ | 5 | Conclusion <br> In context |
| (b) | $\mathrm{H}_{0}: p=0.45 \mathrm{H}_{1}: p>0.45$ | B1 |  | Hypotheses |
|  | $\mathrm{B}(48,0.45)$ | B1 |  | Use of $\mathrm{B}(48,0.45)$ |
|  | $\rightarrow$ Normal mean 21.6 <br> s.d. $\sqrt{ }(48 \times 0.45 \times 0.55)=3.4467$ <br> $($ variance $=11.88)$ | M1 m1 |  | Attempt at normal approx. Method for mean and s.d./variance |
|  | $z=\frac{(30.5-21.6)}{3.4467}=2.58$ | M1 |  | Method for $z$ - ignore sign and incorrect continuity correction |
|  | $\left(\text { or } \frac{(31-21.6)}{3.4467}=2.73\right)$ | A1 |  | $\begin{aligned} & 2.58(2.57 \sim 2.6) \\ & \text { or } 2.73(2.7 \sim 2.75) \end{aligned}$ |
|  | c.v. 1.6449 | B1 |  | 1.6449 ( 1.64~1.65) - ignore sign |
|  | Reject $\mathrm{H}_{0}$ Significant evidence that pass rates have improved following the Easter classes. | A1 | 8 | conclusion - must be compared with correct tail of normal |
| (c)(i) | $90 \%$ confidence interval for mean mark $55.75 \pm 1.6449 \times 9.81 / \sqrt{ } 48$ | $\begin{aligned} & \text { B1 } \\ & \text { M1 } \end{aligned}$ |  | $1.6449 \text { or } 1.678(1.676 \sim 1.679)$ Method for c.i. |
|  | $55.75 \pm 2.329$ $53.4 \sim 58.1$ | A1 |  | 53.4 (53.31~53.45) and <br> 58.1 (58.05~58.15) <br> Allow in $\pm$ form |
| (ii) | Since interval contains 54.2 there is no significant evidence that Easter holiday classes increases mean mark on this test | $\begin{aligned} & \text { E1 } \checkmark \\ & \text { E1 } \checkmark \end{aligned}$ | 5 | no evidence mean mark increased 54.2 in c.i. |


| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 6 cont (d)(i) | Significant evidence of increase in GCSE pass rate but not of increase in mean mark on practice examination. | $\begin{aligned} & \mathrm{E} \sqrt{ } \checkmark \\ & \mathrm{E} 1 \end{aligned}$ | 2 | Their conclusion to (b) or (c) in context <br> Correct conclusions to (b) and (c) stated in context |
| (ii) | The sample was self selecting and so not random. Probably consisting of better motivated students and/or parents so any observed improvements may be due to biased sample rather than the Easter classes. | E1 <br> E1 | 2 | Sample not random <br> Possible reason |
|  | Total |  | 22 |  |
|  | TOTAL |  | 75 |  |


[^0]:    Further copies of this Mark Scheme are available from: aqa.org.uk

