

General Certificate of Education

Statistics 6380

SS04 Statistics 4

Mark Scheme

2006 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method				
m or dM	mark is dependent on one or more M marks and is for method				
A	mark is dependent on M or m marks and is for accuracy				
В	mark is independent of M or m marks and is for method and accuracy				
Е	mark is for explanation				
$\sqrt{\text{or ft or F}}$	follow through from previous				
	incorrect result	MC	mis-copy		
CAO	correct answer only	MR	mis-read		
CSO	correct solution only	RA	required accuracy		
AWFW	anything which falls within	FW	further work		
AWRT	anything which rounds to	ISW	ignore subsequent work		
ACF	any correct form	FIW	from incorrect work		
AG	answer given	BOD	given benefit of doubt		
SC	special case	WR	work replaced by candidate		
OE	or equivalent	FB	formulae book		
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme		
–x EE	deduct x marks for each error	G	graph		
NMS	no method shown	c	candidate		
PI	possibly implied	sf	significant figure(s)		
SCA	substantially correct approach	dp	decimal place(s)		

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

SS04

Q	Solution	Marks	Total	Comments
1(a)	$\bar{x} = 502.22$	B1		502.2 (502 – 502.3)
	s = 2.9486	B1		2.9486 (2.948 – 2.95)
	90% confidence interval	B1		8 df
	$502.22 \pm 1.860 \times \frac{2.9486}{\sqrt{9}}$	B1√		$\sqrt{1.86}$ – their df use of their s.d. / $\sqrt{9}$
	V	M1 m1		correct method – their t
	502.2±1.828 500.39 ~ 504.05	A1		$502.2 (502 - 502.3) \pm 1.828$ $(1.82 - 1.83)$
	300.39 ~ 304.03			or 500.39 (500-500.45)
		B1	8	and 504.05 (504 – 504.1) 1 or 2 dp
			O	•
(b)	Sample from a normal distribution	E1	1	normal
(c)	Since lower limit of confidence interval is above 500g there is significant evidence	E1		mean >500
	that mean is above 500g	E1		some jars < 500
	Some jars contain less than 500g – including two in sample.	E1	3	reason for either
(d)	0.9	B1	1	0.9 CAO or 90%
(u)	Total	Di	13	0.7 CAO 01 7070
2(a)(i)	0.4695	B1	1	0.469 ~ 0.47
(ii)	Poisson, mean $12 \times 2.8 = 33.6$	B1		Poisson, mean 12×2.8 - may be implied
	→ Normal mean 33.6	M1		attempted use of normal approximation
	s.d $\sqrt{33.6} = 5.79655$	m1		$s.d = \sqrt{their mean}$
	$z = \frac{(50.5 - 33.6)}{\sqrt{33.6}} = 2.916$	m1		method for z - ignore sign and continuity corrections
		m1		requires previous M1m1 and attempt at continuity correction
	probability $> 50 = 1 - 0.99823 = 0.0018$	m1		completely correct method
		A1		0.0018 (0.0017 – 0.0019) s.c use of Poisson
				B1 Poisson mean 12×2.8
				M1 method
a > a>	1.26			A1 0.0031 (0.003 – 0.0031)
(b)(i)	$51 \pm \frac{1.96}{\sqrt{51}}$	M1		use of $\sqrt{51}$
	51±14.0	B1		1.96
		m1		allow incorrect z – value and incorrect mean
	37.0 ~ 65.0	A1		$51\pm14.0\ (13.95-14.05)$
				or 37.0 (36.95 – 37.05)
				and 65.0 (64.95 – 65.05)

SS04 (cont)

Q	Solution	Marks	Total	Comments
2(b)(ii)	$\frac{37.0}{12} \sim \frac{65.0}{12}$	M1		their answer to (i)
	3.08 ~ 5.42	A1	2	3.08 (3.08 – 3.1) and 5.42 (5.4 – 5.42) or 4.25±1.17(1.16 – 1.17)
(c)	Evidence arrival rate is greater in the evening as 2.8 is below lower limit of confidence interval.	B1√ E1√	2	greater in evening reason
(d)	Normal used as approximation to Poisson	E1		use of normal
	$\sqrt{51}$ used as approximation to s.d	E1	2	use of approximate s.d
	Total		18	
3(a)	Poisson	B1	1	Poisson can be implied
(b)	$H_0: \mu = 2$	В1		one hypothesis correct – generous & allow $\mu \le 2$
	$H_1: \mu > 2$	В1		both correct – ungenerous
	P(3 or more) = 1 - 0.6767	M1		Attempt to obtain P(3 or more) or any relevant probability if critical values used. Allow wrong tail, normal approx.
	0.323	A1		0.323 $(0.323 \sim 0.324)$ or critical region ≥ 5
	Accept H_0 since $0.323 > 0.1$	A1√		correct conclusion their figures. Disallow for wrong tail or normal approx.
	No significant evidence that mean number of faults exceeds 2	A1√	6	conclusion in context and 0.323 clearly compared with 0.1 or equivalent. Disallow for wrong tail or normal approx.
(c)	Risk of Type 1 error (wrongly claiming mean exceeds 2) can be reduced but this increases the risk of Type 2 error	E1		Risk of Type 2 error increase – generous
	(accepting mean equals 2 when in fact it exceeds 2)	E1	2	in context
	Total		9	

SS04(cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$z = \frac{(21-20)}{2} = 0.5$	M1		Method – generous. Allow 1-0.309 or use 21.5
	2 - 0.3			or use 21.5
	Probability train arrives after 11.00 am is 0.309	A1	2	0.309 (0.308 – 0.309)
(ii)	$z = \frac{(45 - 42)}{3} = 1$	M1		Method – ungenerous
	Probability Ravinder arrives before 11.00am is 0.841	A1	2	0.841 (0.841 – 0.842)
(iii)	$0.30854 \times 0.84134 = 0.260$	M1		Method – their answers to (i) and (ii)
a > a		A1	2	0.260 (0.259 – 0.260)
(b)(i)	Ravinder will arrive before train if $X + 39 > Y + 15$	M1		Any almost correct statement
	X + 39 > 1 + 13 X - Y + 24 > 0	A1	2	Result AG
(ii)	Normal mean $20 - 42 + 24 = 2$	B1 B1		Normal
	Variance $2^2 + 3^2 = 13$ s.d = 3.61	B1	3	13 or 3.61 (3.60 – 3.61) – must know whether variance or s.d. All marks may be implied in part (iii)
(iii)	$z = \frac{(2-0)}{\sqrt{13}} = 0.555$	M1		Method including method for mean and s.d. Allow wrong tail
	probability > 0 is 0.711	A1	2	0.711 (0.708 – 0.713)
(c)	(b)(iii) includes all possibilities. In (a)(iii) Ravinder arrives before train but some possibilities are excluded, e.g. Ravinder arrives 11.01, train arrives 11.02.	E2(1)		Both marks for a clear explanation
	Total		15	
	1000	l		

SS04 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$H_0: p = 0.4$ $H_0: p < 0.4$	B1 B1		one correct hypothesis – generous both hypotheses correct – allow p to imply population
	Binomial $n = 20$ $p = 0.4$	B1		use of binomial
	P (6 or fewer) = 0.250	M1		attempt to calculate P (6 or fewer) or any relevant probability if critical value used. Allow wrong tail on normal approximation
		A1		0.250 (0.2495 – 0.2505) or critical region is 3 or fewer
	Accept H_0 since $0.250 > 0.05$	A 1√		correct conclusion, their figures
	No significant evidence that proportion of passengers rating the meal as poor has been reduced.	A1√	7	In context, 0.05 clearly compared with 0.250 or equivalent. Allow context in (c) Disallow for wrong tail or normal
(b)	$H_0: \mu = 140$	B1		one correct hypothesis – generous
	$H_1: \mu > 140$	B1		both hypotheses correct – ungenerous
	$\bar{x} = 149.4$ s = 12.0296	B1		149.4 (149 or 149.4) and 12.0(12 – 12.05)
	$t = \frac{(149.4 - 140)}{\left(\frac{12.0296}{\sqrt{10}}\right)} = 2.47$	M1		Use of their $\frac{\text{s.d}}{\sqrt{10}}$
	(10)	m1		Correct method for t – ignore sign
	= 2.47	A1		2.47 (2.465 – 2.475)
	c.v t ₉ is 1.833. Reject H ₀	B1		9 df
	significant evidence mean meat content is greater than 140g	B1√ A1√		1.833 or 1.83, their df correct conclusion, their figures – must be compared with correct tail of t. Needs M1m1
		A1√	10	in context, may be earned in (c)
				ALTERNATIVE COMPARISONS
				critical value 146.97 (146.9 to 147) compare with 149.4
				confidence interval 142.43 (142.4 to 142.5 or 142) to 156.37 must compare 140 with 142.43

SS04 (cont)

Q	Solution	Marks	Total	Comments
5(c)	Although less than 40% rated meals as poor the evidence was not significant.	E1		No significant evidence < 40 %
	Convincing evidence that mean meat	E1√		Evidence > 140g – generous
	content is greater than 140g, so second			
	undertaking appears to have been met.	E1	3	some evidence < 40 % e.g disallow for
				'first undertaking not met.'
	Total		20	
	TOTAL		75	