General Certificate of Education January 2006 Advanced Level Examination



# MATHEMATICS Unit Statistics 2A

MS2A/W

Thursday 12 January 2006 1.30 pm to 2.45 pm

#### For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

#### **Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MS2A/W.
- Answer all questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

#### Information

- The maximum mark for this paper is 60.
- The marks for questions are shown in brackets.
- Unit Statistics 2A has a written paper and coursework.

### **Advice**

• Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

## Answer all questions.

1 The administration department at Hitech College uses two photocopiers, A and B, which operate independently.

The number of breakdowns each week, X, for photocopier A can be modelled by a Poisson distribution with a mean of 0.5.

The number of breakdowns each week, Y, for photocopier B can be modelled by a Poisson distribution with a mean of 2.5.

(a) Calculate:

(i) 
$$P(X \le 1)$$
; (2 marks)

(ii) 
$$P(Y = 2)$$
. (2 marks)

- (b) (i) In a given week, show that the probability of a total of exactly 2 photocopier breakdowns is 0.224, correct to three decimal places. (3 marks)
  - (ii) Hence find the probability that there will be exactly 2 photocopier breakdowns in each of four consecutive weeks. (2 marks)
- (c) The total number of photocopier breakdowns, T, in a 4-week period can be modelled by a Poisson distribution with mean  $\mu$ .
  - (i) Find the value of  $\mu$ . (1 mark)
  - (ii) Hence calculate the probability that, in a given 4-week period, there will be a total of at least 18 photocopier breakdowns. (2 marks)
- 2 Year 12 students at Newstatus School choose to participate in one of four sports during the Spring term.

The students' choices are summarised in the table.

	Squash	Badminton	Archery	Hockey	Total
Male	5	16	30	19	70
Female	4	20	33	53	110
Total	9	36	63	72	180

- (a) Use a  $\chi^2$  test, at the 5% level of significance, to determine whether the choice of sport is independent of gender. (10 marks)
- (b) Interpret your result in part (a) as it relates to students choosing hockey. (2 marks)

3 The weight, W grams, of rice contained in a packet can be modelled by a normal distribution with mean  $\mu$  and variance  $\sigma^2$ .

The weights, in grams, of rice contained in each of a random sample of 10 packets were

301.2 305.6 298.4 297.7 292.6 306.1 296.8 299.2 304.0 308.4

- (a) Calculate unbiased estimates for  $\mu$  and  $\sigma^2$ . (2 marks)
- (b) Hence construct a 99% confidence interval for  $\mu$ , giving the limits to one decimal place. (5 marks)
- (c) The packets are claimed to contain, on average, at least 290 grams of rice.

Comment on this claim. (2 marks)

4 (a) A random variable X has probability density function defined by

$$f(x) = \begin{cases} k & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

(i) Show that 
$$k = \frac{1}{b-a}$$
. (1 mark)

(ii) Prove, using integration, that 
$$E(X) = \frac{1}{2}(a+b)$$
. (4 marks)

(b) The error, X grams, made when a shopkeeper weighs out loose sweets can be modelled by a rectangular distribution with the following probability density function:

$$f(x) = \begin{cases} k & -2 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

- (i) Write down the value of the mean,  $\mu$ , of X. (1 mark)
- (ii) Evaluate the standard deviation,  $\sigma$ , of X. (2 marks)

(iii) Hence find 
$$P\left(X < \frac{2-\mu}{\sigma}\right)$$
. (3 marks)

Turn over for the next question

5 The Globe Express agency organises trips to the theatre. The cost, £X, of these trips can be modelled by the following probability distribution:

x	40	45	55	74
P(X=x)	0.30	0.24	0.36	0.10

(a) Calculate the mean and standard deviation of X.

(4 marks)

(b) For special celebrity charity performances, Globe Express increases the cost of the trips to  $\pounds Y$ , where

$$Y = 10X + 250$$

Determine the mean and standard deviation of Y.

(2 marks)

6 Bottles of sherry nominally contain 1000 millilitres. After the introduction of a new method of filling the bottles, there is a suspicion that the mean volume of sherry in a bottle has changed.

In order to investigate this suspicion, a random sample of 12 bottles of sherry is taken and the volume of sherry in each bottle is measured.

The volumes, in millilitres, of sherry in these bottles are found to be

996	1006	1009	999	1007	1003
998	1010	997	996	1008	1007

Assuming that the volume of sherry in a bottle is normally distributed, investigate, at the 5% level of significance, whether the mean volume of sherry in a bottle differs from 1000 millilitres.

(10 marks)

# **END OF QUESTIONS**