

General Certificate of Education
June 2008
Advanced Subsidiary Examination



APPLYING MATHEMATICS
Paper 2

UOM4/2

Friday 16 May 2008 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
- an answer sheet for use in Questions 2 and 4 (enclosed)
- a graphics calculator
- a ruler.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book **and** on the top of the answer sheet for Questions 2 and 4. The *Examining Body* for this paper is AQA. The *Paper Reference* is UOM4/2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should normally be given to three significant figures.
- At the end of the examination, remember to hand in both your answer book **and** the answer sheet for Questions 2 and 4.

Information

- The maximum mark for this paper is 70.
- The marks for questions are shown in brackets.
- You will be awarded up to 3 marks for your ability to present information accurately using correct notation **and** up to 3 marks for mathematical arguments presented clearly and logically.

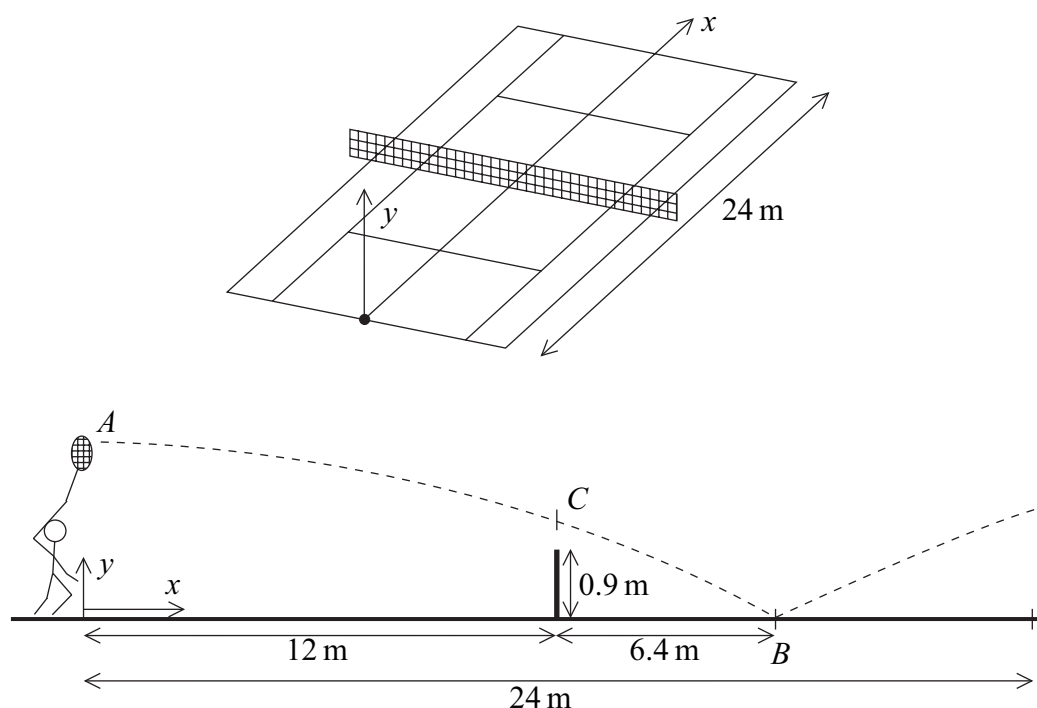
SECTION A

Answer **all** questions.

- 1 The position of a tennis ball t seconds after it is served by a player can be modelled by two functions:

- the horizontal distance, x metres, that the ball has travelled is given by $x = 27.5t$;
- the vertical distance of the ball above the horizontal ground, y metres, is given by $y = h - 5t^2$, where h metres is the height above the ground at which the ball is hit.

The diagrams show some of the dimensions of a tennis court.



Assume that this particular player serves the ball from point A and that the ball first hits the ground at point B . The point C is vertically above the net.

- (a) (i) Show that it will take the ball 0.669 seconds to travel 18.4 metres horizontally from point A to point B . (2 marks)
- (ii) When the ball hits the ground at point B , $y = 0$. Use the information given in part (a)(i) to find the height at which the ball was hit. (2 marks)
- (iii) By considering the horizontal distance travelled by the ball, find how long it will take the ball to travel from A to C . (2 marks)

(b) Another player stands in the same position and serves a tennis ball from a height of 3 metres above the ground. For this serve, two different functions can be used to model the position of the tennis ball, t seconds after it is served:

- the horizontal distance, x metres, that the ball has travelled is given by $x = 32.5t$;
- the vertical distance of the ball above the horizontal ground, y metres, is given by $y = 3 - 5t^2$.

- (i) Find how long it takes the ball to hit the ground. *(3 marks)*
- (ii) Show that the ball lands outside of the court. *(3 marks)*
- (iii) Sketch a graph showing y plotted against t , showing clearly all significant features. *(3 marks)*

Turn over for the next question

SECTION B

Answer **all** questions.

- 2 Customers can hire a barge from a barge hire company. The charge per week is reduced if the barge is hired for more than one week.

Given that $\pounds H_n$ is the hire charge in the n th week, then $\pounds H_{n+1}$ is the hire charge in the next week, where $H_{n+1} = 0.9H_n - 25$.

The recurrence relation can be used to find the cost of hiring a barge in successive weeks. The cost of hiring a barge is $\pounds 1000$ for the first week.

- (a) Show clearly that the cost of hiring a barge in the second week is $\pounds 875$. (2 marks)
- (b) Find the total cost of hiring a barge for four weeks. (3 marks)

The barge hire company has two centres on a canal network from which customers can hire barges.

Assume that:

- all barges are hired for just one week;
- some customers hire barges and travel from one centre to the other and leave the barges there;
- all remaining customers return their barges to the centres from which they hired them.

Let A_n be the number of barges at centre A at the end of the n th week, and B_n be the number of barges at centre B at the end of the n th week.

Assume that 80% of the barges hired from centre A are returned there and that the remainder are left at centre B at the end of the week.

The number of barges in each centre at the end of the n th week can be modelled by the recurrence relations

$$A_n = 0.8A_{n-1} + 0.1B_{n-1}$$

$$B_n = 0.9B_{n-1} + 0.2A_{n-1}$$

- (c) According to these recurrence relations, state:
- (i) the percentage of barges from centre A that are left at centre B at the end of a week; (1 mark)
- (ii) the percentage of barges from centre B that are left at centre A at the end of a week. (1 mark)

There are initially 200 barges in centre A and 100 barges in centre B (that is, $A_0 = 200$ and $B_0 = 100$).

- (d) Show calculations that confirm that at the end of the first week there are 170 barges in centre A and 130 barges in centre B . *(2 marks)*
- (e) Complete the table on the answer sheet, showing the number of barges that the recurrence relations predict will be in each centre up to the end of the third week. *(4 marks)*
- (f) Suggest one way in which these recurrence relations do not accurately model the situation. *(2 marks)*

Turn over for the next question

SECTION C

Answer **all** questions.

- 3 The number of hours after midnight at which sunrise occurs, T_{sunrise} , in London throughout 2008 can be modelled by the equation $T_{\text{sunrise}} = 6 + 2 \cos n^\circ$, where n is the number of the day in the year ($n = 0$ on January 1st). For example, when $n = 90$, $T_{\text{sunrise}} = 6 + 2 \cos(90)^\circ = 6$, so sunrise is at 6 am.
- (a) Find the time of sunrise on January 31st. Give your answer in hours and minutes. (3 marks)
- (b) (i) State the minimum value of T_{sunrise} given by the equation. (1 mark)
- (ii) State the value of n when this minimum occurs. (1 mark)
- (iii) Interpret your answers to parts (b)(i) and (b)(ii) in terms of sunrise times. (1 mark)
- (c) (i) State the maximum value of T_{sunrise} given by the equation. (1 mark)
- (ii) State the values of n when this maximum occurs. (2 marks)
- (iii) Interpret your answers to parts (c)(i) and (c)(ii) in terms of sunrise times. (1 mark)
- (d) Sketch a graph showing sunrise times in London throughout 2008 plotted against n for $0 \leq n \leq 360$. (2 marks)
- (e) Find the values of n for which sunrise is at 7 am. (4 marks)

SECTION D

Answer **all** questions.

- 4 A budget airline operates a flight from Manchester to Rome. Seats for a flight are first on sale 6 months before the date of the flight and the price, £ P , of a seat changes at the beginning of each month according to the table below.

Number of complete months before flight	Price of seat	
	Winter	Summer
5	£5.00	£10.00
4	£7.50	£12.50
3	£10.00	£15.00
2	£12.50	£20.00
1	£20.00	£25.00
0	£30.00	£50.00

An airline manager carries out a simulation to estimate how much income will be generated by sales. She assumes that in any one month 10, 20 or 30 seats will be sold.

The table shows how she assigns randomly generated integers to simulate sales in each month for winter and summer flights.

Number of seats sold	Winter flights	Summer flights
10	0, 1	9
20	2, 3, 4, 5, 6	5, 6, 7, 8
30	7, 8, 9	0, 1, 2, 3, 4

- (a) (i) Write down the probability that for a winter flight 10 seats will be sold in a month. (1 mark)
- (ii) Explain how you deduced your answer. (1 mark)

Question 4 continues on the next page

- (b) For one winter flight, the airline manager carries out two trials of a simulation using the random numbers in **Table 1** on the answer sheet to predict the number of seats sold in each month. She then uses these predictions to estimate the amount of income which the airline receives for sales of seats for this winter flight.

Complete **Table 1** on the answer sheet for these two winter trials. *(8 marks)*

- (c) Comment on the results of these two different trials. *(2 marks)*

- (d) For summer flights, the airline manager decides that the airline will add an extra charge of £25 to the price of every seat after the first 100 seats have been sold.

The manager carries out a single trial of a simulation using the random numbers in **Table 2** on the answer sheet.

Complete this table on the answer sheet. *(5 marks)*

- (e) Suggest one way in which the simulations could be improved. *(1 mark)*

END OF QUESTIONS

Surname						Other Names					
Centre Number						Candidate Number					
Candidate Signature											

For Examiner's Use

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UOM4/2AS



This answer sheet is to be used when answering Questions 2 and 4, as indicated.

Fasten this sheet securely to your answer book.

This table is to be used when answering Question 2.

Week	Number of barges in Centre A	Number of barges in Centre B
0	200	100
1	170	130
2		
3		

These tables are to be used when answering Question 4.

Table 1 Winter flights

No. of complete months before flight	Price of seat	TRIAL 1			TRIAL 2		
		Random number	Number of seats sold	Income	Random number	Number of seats sold	Income
5	£5.00	7			9		
4	£7.50	4			7		
3	£10.00	5			2		
2	£12.50	1			9		
1	£20.00	9			3		
0	£30.00	9			0		
		Total			Total		

Table 2 Summer flights

No. of complete months before flight	Price of seat			
		Random number	Number of seats sold	Income
5	£10.00	9		
4	£12.50	7		
3	£15.00	2		
2	£20.00	4		
1	£25.00	3		
0	£50.00	0		
		Total		

END OF ANSWER SHEET

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