

General Certificate of Education  
June 2008  
Advanced Subsidiary Examination



**APPLYING MATHEMATICS**  
**Paper 1**

**UOM4/1**

Friday 16 May 2008 9.00 am to 10.00 am

**For this paper you must have:**

- an 8-page answer book
- a clean copy of the Data Sheet (enclosed)
- a graphics calculator
- a ruler.

Time allowed: 1 hour

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is UOM4/1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- You may **not** refer to the copy of the Data Sheet that was available prior to this examination. A clean copy is available for your use.

**Information**

- The maximum mark for this paper is 30.
- The marks for questions are shown in brackets.

Answer **all** questions.

Use **Population growth** on the Data Sheet.

- 1 A sum of money is invested and receives compound interest. The growth can be expressed by the recurrence relation  $P_{n+1} = 1.025P_n$ , where  $P_n$  is the amount of money at the end of the  $n^{\text{th}}$  year, and  $P_{n+1}$  is the amount of money one year later.

- (a) (i) State the interest rate as a percentage in this case. (1 mark)
- (ii) Make a copy of the table below.

$n$ , year number	$P_n$ , amount at end of $n^{\text{th}}$ year
0	£200.00
1	£205.00
2	
3	
4	
5	

Complete your table using the recurrence relation  $P_{n+1} = 1.025P_n$  to show how much an initial investment of £200 is worth at the end of each successive year for the first 5 years.

You should give values to the nearest penny, requiring you to work with values correct to at least three decimal places. (2 marks)

- (b) This growth can also be modelled by the exponential function  $P = P_0 e^{kt}$ , where  $P$  is the value of the investment after  $t$  years,  $P_0$  is the initial investment and  $k$  is a constant.
- (i) Sketch a graph of  $P$  plotted against  $t$  that shows clearly all significant features. (2 marks)
- (ii) Find  $k$ , using the method of considering growth in the first year, as suggested in the article. (3 marks)

- 2 The article explains that for a population that doubles every 25 years, ' $2P_0 = P_0 \times e^{25k}$  leading to  $k = 0.0277$ '.

Show clearly how this value of  $k$  can be obtained. (3 marks)

3 Scotland currently has a population growth rate of 0.4% .

(a) Use the ‘rule of 70’ to find how long it will take the population to double in size. (2 marks)

(b) Suggest a reason why, in reality, this is unlikely to be the case. (1 mark)

4 The data in **Figure 2** show that the population of the world approximately doubled between 1960 and 2000. The ‘rule of 70’ gives the annual percentage growth rate over this period as approximately 1.75% .

(a) Use this growth rate together with  $(1 + r) = e^k$  to confirm that the value of  $k$  in the exponential model is given by  $k = 0.0173$  . (3 marks)

(b) (i) The exponential model

$$P = 3e^{0.0173(t-1960)}$$

predicts the population in the calendar year  $t$ .

Find the population this predicts for the year 2050. (2 marks)

(ii) Explain why this value is not the same as the value suggested in the table of **Figure 2** in the article. (2 marks)

5 Find a linear model for the population,  $P$  billions of people, of the world in year  $t$ , assuming that the population of the world was 3 billion in 1960 and 6 billion in 2000. (4 marks)

6 Use the recurrence relation

$$P_{n+1} = P_n + 0.035P_n \left(1 - \frac{P_n}{9.8}\right)$$

and a value of the world’s population  $P_0 = 3$  (billion) in 1960 to find the population of the world in 1961. (3 marks)

7 (a) State the limiting value of the world’s population according to the logistic model

$$P_{n+1} = P_n + 0.035P_n \left(1 - \frac{P_n}{9.8}\right) \quad (1 \text{ mark})$$

(b) Explain, making reference to the logistic model, why you chose this value. (1 mark)

**There are no questions printed on this page**