

# Free-Standing Mathematics Qualification June 2011 

Mathematics Advanced Level 6992
(Specification 6992)
Modelling with Calculus

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## Key to mark scheme and abbreviations used in marking

| M | mark is for method |  |
| :--- | :--- | :--- |
| $m$ or dM | mark is dependent on one or more M marks and is for method |  |
| A | mark is dependent on M or m marks and is for accuracy |  |
| B | mark is independent of M or m marks and is for method and accuracy <br> E <br> mark is for explanation |  |
| Jor ft or F | follow through from previous <br> incorrect result |  |
| CAO | correct answer only | MC |

## Application of Mark Scheme

## No method shown:

Correct answer without working
Incorrect answer without working

## More than one method / choice of solution:

2 or more complete attempts, neither/none crossed out
1 complete and 1 partial attempt, neither crossed out

## Crossed out work

Alternative solution using a correct or partially correct method
mark as in scheme zero marks unless specified otherwise
mark both/all fully and award the mean mark rounded down award credit for the complete solution only
do not mark unless it has not been replaced
award method and accuracy marks as appropriate

## Free-Standing Mathematics Qualification

Modelling with Calculus (6992)
Answers and Marking Scheme - June 2011

## Question 1

| (a) | $\frac{\mathrm{d} h}{\mathrm{~d} t}=32 t-193$ <br> $\frac{\mathrm{~d} h}{\mathrm{~d} t}=0 \Rightarrow$ <br> $32 t-193=0$ <br> $t=\frac{193}{32}$ or 6.03 <br> When $t=\frac{193}{32}$, <br> $h=16\left(\frac{193}{32}\right)^{2}-193\left(\frac{193}{32}\right)+780$ <br> $=198$ | M 1 A 1 | M1 2 terms, either correct |
| :---: | :--- | :---: | :--- |
|  | M1 |  |  |
| (b)(i) | Quadratic shape with minimum point <br> Cuts $h$ axis at 780 and goes higher as $\mathrm{t} \rightarrow 18$ | M 1 | SC5 for 197 |
| (ii) | The model is not appropriate for values of <br> $t>12$ and reason why | E 1 | SC1 for plot not sketch |
|  | TOTAL | $\mathbf{9}$ |  |

## Question 2

| (a) | $\begin{aligned} & \frac{\mathrm{d} v}{\mathrm{~d} t}=-2+10 t-6 t^{2} \\ & \frac{\mathrm{~d} v}{\mathrm{~d} t}=0 \Rightarrow \\ & -2+10 t-6 t^{2}=0 \\ & 3 t^{2}-5 t+1=0 \\ & t=\frac{5 \pm \sqrt{25-12}}{6} \\ & =\frac{8.606}{6} \text { or } \frac{1.394}{6} \\ & =1.43 \text { or } 0.232 \end{aligned}$ | M1A1 <br> M1 <br> M1 <br> A1 | M1 2 terms correct; could be seen in (b) <br> SC4 either answer correct Accept 2 dp |
| :---: | :---: | :---: | :---: |
| (b) | $\frac{\mathrm{d}^{2} v}{\mathrm{~d} t^{2}}=10-12 t$ | M1A1ft | M1 Either term correct $\mathrm{ft} \frac{\mathrm{d} v}{\mathrm{~d} t}$ |
| (c) | When $v=1.43$, $\begin{aligned} & v=16-2 \times 1.43+5(1.43)^{2}-2(1.43)^{3} \\ & =17.516 \end{aligned}$ <br> Maximum value is $£ 17.52$ <br> When $t=1.43$, $\frac{\mathrm{d}^{2} v}{\mathrm{~d} t^{2}}=-7.211$ <br> This is negative, hence answer is a maximum | M1 <br> A1 <br> B1 <br> E1 | Condone 17.5 |
| (d) | $\begin{aligned} & \frac{\mathrm{d}^{2} v}{\mathrm{~d} t^{2}}=0 \text { when } 10-12 t=0 \\ & t=\frac{5}{6} \text { or } 0.833 \end{aligned}$ <br> The value of the shares is increasing at its fastest rate | M1 <br> A1 <br> E1 |  |
|  | TOTAL | 14 |  |

## Question 3

| (a)(i) | Three strips <br> $\Rightarrow$ values of $t$ are $0,1,2$ and 3 <br> When $t=0, v=16$ $\begin{aligned} & t=1, v=17 \\ & t=2, v=16 \\ & t=3, v=1 \end{aligned}$ <br> Area $\begin{aligned} & \approx \frac{1}{2} \times 1\{16+1+2(17+16) \\ & =\frac{1}{2}(17+2 \times 33) \\ & =41.5 \end{aligned}$ | B2 <br> M1A1 <br> A1 | B1 for any 2 correct <br> Ignore subsequent $\div 3$ |
| :---: | :---: | :---: | :---: |
| (ii) | Use more strips | E1 | Or use the integral |
| (b) | $\begin{aligned} & \int_{0}^{3}\left(16-2 t+5 t^{2}-2 t^{3}\right) \mathrm{d} t \\ & =\left[16 t-t^{2}+\frac{5}{3} t^{3}-\frac{1}{2} t^{4}\right]_{0}^{3} \\ & =48-9+45-40.5 \\ & =43.5 \end{aligned}$ | $\begin{gathered} \text { B1 } \\ \text { B1 } \\ \text { B1 } \\ \text { M1 } \\ \text { A1 } \end{gathered}$ | SC4 use of 1.6 or $1.7 t^{3}$ <br> B1 2 terms correct <br> B2 3 terms correct <br> B3 4 terms correct <br> 2 terms correct and limits |
| (c) | $\therefore$ Mean value is $£ \frac{43.5}{3}$ $=£ 14.50$ | M1 <br> A1 ft | Accept $41.5 \div 3=13.83$ <br> ft (a)(i) or (b) <br> Condone $\frac{(\mathrm{a})(\mathrm{i})+(\mathrm{b})}{2} \div 3$ |
|  | TOTAL | 13 |  |

## Question 4

| (a) | When $x=6, h=70+40 \cos 2 \pi$ $\begin{aligned} & =70+40 \times 1 \\ & =110 \end{aligned}$ | $\begin{aligned} & \text { B1 } \\ & \text { B1 } \end{aligned}$ | $\begin{aligned} & \text { B1 for } \cos 2 \pi=1 \\ & 109.9 \sim 110 \mathrm{SC} 1 \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| (b) | $\frac{\mathrm{d} h}{\mathrm{~d} x}=-40 \cdot \frac{\pi}{3} \sin \frac{\pi}{3} x$ | B1 B1 B1 | $\begin{aligned} & \frac{\pi}{3} \\ & \sin \frac{\pi}{3} x \end{aligned}$ <br> All correct |
| (c) | Maximum value of $-\sin \frac{\pi}{3} x$ is 1 <br> Maximum value is $40 . \frac{\pi}{3}=41.88 \ldots$ $=41.9 \text { or } \frac{40 \pi}{3}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { A1 } \end{aligned}$ | Condone max value of $\sin \frac{\pi}{3} x$ is 1 $-41.9 \mathrm{SC} 2$ |
|  | TOTAL | 8 |  |

## Question 5

| (a) | $\begin{aligned} & \frac{\mathrm{d} m}{\mathrm{~d} t}=-k m \\ & \int \frac{\mathrm{~d} m}{m}=-\int k d t \\ & \ln m=-\mathrm{k} t+c \\ & m=\mathrm{Ce}^{-k t} \end{aligned}$ | $\begin{gathered} \text { M1 } \\ \text { A1 A1 } \\ \text { M1 } \\ \text { A1 } \end{gathered}$ | Need this line for A2 <br> M1 for + c <br> Can be obtained in (b) <br> Need correct working |
| :---: | :---: | :---: | :---: |
| (b) | When $\mathrm{t}=0, \mathrm{~m}=40, \therefore \mathrm{C}=40$ $m=40 \mathrm{e}^{-k t}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |  |
| (c) | When $\mathrm{t}=6,20=40 \mathrm{e}^{-k 6}$ $\begin{aligned} & \mathrm{e}^{-6 k}=\frac{1}{2} \\ & \ln \left(\frac{1}{2}\right)=-6 k \\ & -6 \mathrm{k}=-0.693147 \\ & k=0.1155 \\ & =0.116 \end{aligned}$ | M1 <br> A1 <br> A1 | Or $\ln 2=6 \mathrm{k}$ <br> (M1 A1 for $-k=-0.116$ ) <br> Accept 0.115 <br> SC1-0.116 |
| (d) | $\begin{aligned} & \text { When } t=18, m=40 \mathrm{e}^{-18 k} \\ & =5.002 \ldots \\ & =5 \end{aligned}$ | B1 <br> B1 <br> B1 | Condone $4 . \dot{9}$ <br> Accept 5.000 ... or 4.96 to 5 <br> Exact answer (not rounded) <br> NB Three 'half-lives' hence mass is 5 grams SC3 |
| (e) | $\begin{aligned} & \text { When } \mathrm{m}=2, \quad 2=40 e^{-k t} \\ & \ln 0.05=-0.1155 \mathrm{t} \\ & t=25.9 \end{aligned}$ | M1 <br> A1 <br> A1 | or $6 \ln 20 / \ln 2$ <br> Condone 25.8 or 26 |
|  | TOTAL | 16 |  |
|  | TOTAL MARK FOR PAPER | 60 |  |


[^0]:    Further copies of this Mark Scheme are available from: aqa.org.uk

