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Free-Standing Mathematics Qualification June 2011

Mathematics Advanced Level

6992

(Specification 6992)

Modelling with Calculus



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Key to mark scheme and abbreviations used in marking

mark is for method			
mark is dependent on one or more M marks and is for method			
mark is dependent on M or m marks and is for accuracy			
mark is independent of M or m marks and is for method and accuracy			
mark is for explanation			
follow through from previous			
incorrect result	MC	mis-copy	
correct answer only	MR	mis-read	
correct solution only	RA	required accuracy	
anything which falls within	FW	further work	
anything which rounds to	ISW	ignore subsequent work	
any correct form	FIW	from incorrect work	
answer given	BOD	given benefit of doubt	
special case	WR	work replaced by candidate	
OE	FB	formulae book	
2 or 1 (or 0) accuracy marks	NOS	not on scheme	
deduct <i>x</i> marks for each error	G	graph	
no method shown	c	candidate	
possibly implied	sf	significant figure(s)	
substantially correct approach	dp	decimal place(s)	
	mark is for method mark is dependent on one or me mark is dependent on M or m n mark is independent of M or m mark is for explanation follow through from previous incorrect result correct answer only correct solution only anything which falls within anything which falls within anything which rounds to any correct form answer given special case OE 2 or 1 (or 0) accuracy marks deduct <i>x</i> marks for each error no method shown possibly implied substantially correct approach	mark is for methodmark is dependent on one or more M marks andmark is dependent on M or m marks and is formark is independent of M or m marks and is formark is for explanationfollow through from previousincorrect resultMCcorrect answer onlyMRcorrect solution onlyRAanything which falls withinFWanything which rounds toISWany correct formFIWanswer givenBODspecial caseWROEFB2 or 1 (or 0) accuracy marksNOSdeduct x marks for each errorGno method showncpossibly impliedsfsubstantially correct approachdp	

Application of Mark Scheme

mark as in scheme
zero marks unless specified otherwise
mark both/all fully and award the mean mark rounded down
award credit for the complete solution only
do not mark unless it has not been replaced
award method and accuracy marks as appropriate

Free-Standing Mathematics Qualification Modelling with Calculus (6992) Answers and Marking Scheme - June 2011

(a)	$\frac{\mathrm{d}h}{\mathrm{d}t} = 32t - 193$	M1A1	M1 2 terms, either correct
	$\frac{\mathrm{d}t}{\mathrm{d}t} = 0 \Longrightarrow$		
	32t - 193 = 0	M1	
	$t = \frac{193}{32}$ or 6.03	A1	
	When $t = \frac{193}{32}$,		
	$h = 16\left(\frac{193}{32}\right)^2 - 193\left(\frac{193}{32}\right) + 780$	M1	SC5 for 197
	= 198	A1	Use of $t = 6$ only etc SC5
(b)(i)	Quadratic shape with minimum point	B1	
	Cuts <i>h</i> axis at 780 and goes higher as $t \rightarrow 18$	B1	SC1 for plot not sketch
(ii)	The model is not appropriate for values of $t > 12$ and reason why	E1	
	TOTAL	9	

(a)	$\frac{\mathrm{d}v}{\mathrm{d}t} = -2 + 10t - 6t^2$	M1A1	M1 2 terms correct; could be seen in (b)
	$\frac{\mathrm{d}v}{\mathrm{d}t} = 0 \Longrightarrow$	M1	
	$-2 + 10t - 6t^2 = 0$		
	$3t^2 - 5t + 1 = 0$		
	$t = \frac{5 \pm \sqrt{25 - 12}}{6}$	M1	
	$=\frac{8.606}{6}$ or $\frac{1.394}{6}$		
	= 1.43 or 0.232	A1	SC4 either answer correct Accept 2 dp
	$d^2 y$		M1 Either term correct
(b)	$\frac{dt}{dt^2} = 10 - 12t$	M1A1ft	ft $\frac{\mathrm{d}v}{\mathrm{d}t}$
(c)	When $v = 1.43$,	M1	
	$v = 16 - 2 \times 1.43 + 5(1.43)^2 - 2(1.43)^3$		
	= 17.516		
	Maximum value is £17.52	A1	Condone 17.5
	When $t = 1.43$,		
	$\frac{\mathrm{d}^2 v}{\mathrm{d}t^2} = -7.211$	B1	
	This is negative, hence answer is a maximum	E1	
(d)	$\frac{d^2 v}{dt^2} = 0$ when $10 - 12t = 0$	M1	
	$t = \frac{5}{6}$ or 0.833	A1	
	The value of the shares is increasing at its fastest rate	E1	
	TOTAL	14	

(a)(i)	Three strips		
(a)(1)	\Rightarrow values of <i>t</i> are 0, 1, 2 and 3		
	When $t = 0, v = 16$	B2	B1 for any 2 correct
	t = 1, v = 17		
	t = 2, v = 16		
	t = 3, v = 1		
	Area		
	$\approx \frac{1}{2} \times 1\{16+1+2 \ (17+16)$	M1A1	
	$=\frac{1}{2}(17+2\times 33)$		
	= 41.5	A1	Ignore subsequent ÷3
(ii)	Use more strips	E1	Or use the integral
(b)	$\int_0^3 (16 - 2t + 5t^2 - 2t^3) dt$		SC4 use of 1.6 or $1.7t^3$
	$\begin{bmatrix} 1 & 2 & 5 & 3 & 1 \end{bmatrix}^3$	B1	B1 2 terms correct
	$= \left 16t - t^2 + \frac{1}{3}t^3 - \frac{1}{2}t^4 \right _{0}$	B1	B2 3 terms correct
		BI	B3 4 terms correct
	=48-9+45-40.5	MI	2 terms correct and limits
	= 43.5	A1	
(c)	\therefore Mean value is $\pounds \frac{43.5}{3}$	M 1	Accept $41.5 \div 3 = 13.83$
			ft (a)(i) or (b)
	= £ 14.50	A1 ft	Condone $\frac{(a)(i) + (b)}{2} \div 3$
	TOTAL	13	

Question	4
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(a)	When $x = 6$, $h = 70 + 40 \cos 2\pi$		
	$= 70 + 40 \times 1$	B1	B1 for $\cos 2\pi = 1$
	= 110	B1	109.9 ~ 110 SC1
		B1	<u>π</u>
	dh α π π		3
(b)	$\frac{dx}{dx} = -40.\frac{dx}{3}\sin\frac{dx}{3}x$	B 1	$\sin\frac{\pi}{3}x$
		B1	All correct
	π		Condone
(c)	Maximum value of $-\sin\frac{\pi}{3}x$ is 1	M1	max value of $\sin \frac{\pi}{3}x$ is 1
	Maximum value is $40.\frac{\pi}{2} = 41.88$	A1	
	3		
	$=41.9 \text{ or } \frac{40\pi}{3}$	A1	- 41.9 SC2
	TOTAL	8	

(a)	$\frac{\mathrm{d}m}{\mathrm{d}t} = -k \ m$		
	$\int \frac{\mathrm{d}m}{m} = -\int k dt$	M1	
	$\ln m = -\mathbf{k} t + c$	A1 A1 M1	Need this line for A2 M1 for + c
	$m = \mathrm{C} \mathrm{e}^{-kt}$	A1	Can be obtained in (b) Need correct working
(b)	When $t = 0$, $m = 40$, $\therefore C = 40$	M1	
	$m = 40 e^{-kt}$	A1	
(c)	When $t = 6$, $20 = 40 e^{-k6}$		
	$e^{-6k} = \frac{1}{2}$		
	$\ln\left(\frac{1}{2}\right) = -6k$	M1	Or $\ln 2 = 6k$
	-6 k = -0.693147	A1	
	k = 0.1155		(M1 A1 for $-k = -0.116$)
	= 0.116	A1	Accept 0.115 SC1 – 0.116
(d)	When $t = 18$, $m = 40 e^{-18k}$	B1	Condone 4.9
	= 5.002	B1	Accept 5.000or 4.96 to 5
	= 5	B 1	Exact answer (not rounded)
			NB Three 'half-lives' hence mass is 5 grams SC3
(e)	When m = 2, 2 = 40 e^{-kt}	M1	
	$\ln 0.05 = -0.1155 \text{ t}$	A1	or 6 ln20/ln2
	<i>t</i> = 25.9	A1	Condone 25.8 or 26
	TOTAL	16	
	TOTAL MARK FOR PAPER	60	