Centre Number						Candidate Number				
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Other Names										
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For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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5	
6	
TOTAL	

Free-Standing Mathematics Qualification
Advanced Level
June 2013

Modelling with Calculus 6992/2

Unit 12

Friday 17 May 2013 9.00 am to 10.30 am

For this paper you must have:
- a clean copy of the Data Sheet (enclosed)
- a calculator
- a ruler.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should normally be given to three significant figures, unless stated otherwise.
- You may **not** refer to the copy of the Data Sheet that was available prior to this examination. A clean copy is available for your use.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 60.
- You may use either a scientific calculator or a graphics calculator.

Advice
- You do not necessarily need to use all the space provided.

Section A

Answer **all** questions.

Answer each question in the space provided for that question.

Use **Cricket** *on page 2 of the Data Sheet.*

1 Freddie hits a ball while playing cricket.

The height of the ball, h metres, above A, the point at which it was hit, is given by

$$h = 12t - 5t^2$$

where t is the time in seconds after the ball is hit.

1 (a) Find the height of the ball above A when $t = 1$. *(1 mark)*

1 (b) Find $\dfrac{\mathrm{d}h}{\mathrm{d}t}$. *(2 marks)*

1 (c) Find t when $\dfrac{\mathrm{d}h}{\mathrm{d}t} = 0$. *(2 marks)*

1 (d) Hence predict the maximum height of the ball above A. *(2 marks)*

1 (e) (i) Find $\dfrac{\mathrm{d}^2h}{\mathrm{d}t^2}$. *(1 mark)*

1 (e) (ii) Hence state how this value confirms that the answer to part **(d)** is the maximum height and not the minimum height. *(1 mark)*

QUESTION PART REFERENCE

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Section B

Answer **all** questions.

Answer each question in the space provided for that question.

Use **Traffic flow** *on page 3 of the Data Sheet.*

2 The number of vehicles per hour, n, using the trunk roads in non built-up areas, can be modelled by

$$n = 50t^2 - t^4 + 4000$$

for $-6 \leqslant t \leqslant 6$, where t is the number of hours after 1 pm.

Use this model and calculus to answer the following questions.

2 (a) Find $\dfrac{dn}{dt}$. *(2 marks)*

2 (b) Hence find the values of t at the stationary points of

$$n = 50t^2 - t^4 + 4000$$

You may use the factorisation $at - bt^3 = t(a - bt^2)$. *(4 marks)*

2 (c) Find $\dfrac{d^2n}{dt^2}$. *(2 marks)*

2 (d) Use your answer to part **(c)** to find which values of t found in part **(b)** give a maximum value for n. *(2 marks)*

2 (e) Find the maximum number of vehicles per hour predicted by this model.

At what times of day is the number of vehicles a maximum? *(3 marks)*

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3 The total number of vehicles using the trunk roads in non built-up areas in Surrey between 7 am and 7 pm, during an average weekday, may be modelled as

$$\int_{-6}^{6} (50t^2 - t^4 + 4000)\, dt$$

3 (a) Use the trapezium rule with four strips to find an estimate for the total number of vehicles using these roads in Surrey between 7 am and 7 pm during an average weekday. *(5 marks)*

3 (b) (i) Use integration to find the value of

$$\int_{-6}^{6} (50t^2 - t^4 + 4000)\, dt$$ *(4 marks)*

3 (b) (ii) Hence give an estimate of the average number of vehicles per hour using these roads in Surrey between 7 am and 7 pm. *(2 marks)*

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Section C

Answer **all** questions.

Answer each question in the space provided for that question.

Use **Fridge** *on page 4 of the Data Sheet.*

4 A can of cola is at a temperature of 23 °C.
The can of cola is placed in a fridge which has a temperature of 3 °C.
After t minutes, the temperature, c (in °C), of the can of cola satisfies the equation

$$\frac{\mathrm{d}c}{\mathrm{d}t} = -\frac{1}{25}(c-3)$$

4 (a) (i) Find $\frac{\mathrm{d}c}{\mathrm{d}t}$ when $c = 20$. *(1 mark)*

4 (a) (ii) Interpret this value. *(1 mark)*

4 (b) Show that $\frac{1}{25}t = \ln\frac{20}{c-3}$. *(4 marks)*

4 (c) Wendy would like the temperature of the can of cola to be 5 °C.
Find the value of t when the can of cola is at this temperature. *(2 marks)*

4 (d) Using part **(b)**, show that

$$c = 3 + 20\mathrm{e}^{-\frac{1}{25}t}$$

(3 marks)

4 (e) Find the temperature of the can of cola after 6 minutes. *(2 marks)*

4 (f) (i) State the value which c approaches as t becomes very large. *(1 mark)*

4 (f) (ii) State the value of $\frac{\mathrm{d}c}{\mathrm{d}t}$ as t becomes very large. *(1 mark)*

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5 At the same time as she placed the can of cola in the fridge, Wendy also placed a bottle of water in a freezer.
After t minutes, the temperature of the bottle of water, w (in °C), can be modelled by the equation

$$w = -7 + 30\mathrm{e}^{-\frac{1}{2}t}$$

Find w when

5 (a) (i) $t = 2$.

5 (a) (ii) $t = 2.1$.

Give your answers to five decimal places. *(2 marks)*

5 (b) Using the numerical approximation $\dfrac{f(a+h)-f(a)}{h}$ and your answers to part **(a)**, find an estimate for $\dfrac{\mathrm{d}w}{\mathrm{d}t}$ when $t = 2$. *(2 marks)*

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Section D

Answer **all** questions.

Answer each question in the space provided for that question.

Use **Birds in a garden** *on page 4 of the Data Sheet.*

6 The total number of birds, n, seen by Paolo in his garden each month can be modelled by

$$n = 110 + 50\sin\frac{\pi}{6}t$$

where t is the number of months after Paolo starts his recording.

6 (a) Find the number of birds predicted by the model after three months. *(2 marks)*

6 (b) (i) Show that the model predicts that the number of birds is a minimum when $t = 9$. *(2 marks)*

6 (b) (ii) Find when the model predicts the next minimum point. *(2 marks)*

6 (c) Find an expression for $\dfrac{dn}{dt}$. *(2 marks)*

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