General Certificate of Education
January 2008
Advanced Level Examination

MATHEMATICS **MS2B**
Unit Statistics 2B

Friday 11 January 2008 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MS2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.
- Unit Statistics 2B has a **written paper only**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 David claims that customers have to queue at a supermarket checkout for more than 5 minutes, on average.

The queuing times, x minutes, of 40 randomly selected customers result in $\bar{x} = 5.5$ and $s^2 = 1.31$.

Investigate, at the 1% level of significance, David's claim. *(6 marks)*

2 A new information technology centre is advertising places on its one-week residential computer courses.

(a) The number of places, X, booked each week on the publishing course may be modelled by a Poisson distribution with a mean of 9.0.

(i) State the standard deviation of X. *(1 mark)*

(ii) Calculate $\mathrm{P}(6 < X < 12)$. *(3 marks)*

(b) The number of places booked each week on the web design course may be modelled by a Poisson distribution with a mean of 2.5.

(i) Write down the distribution for T, the **total** number of places booked each week on the publishing and web design courses. *(1 mark)*

(ii) Hence calculate the probability that, during a given week, a total of fewer than 2 places are booked. *(3 marks)*

(c) The number of places booked on the database course during each of a random sample of 10 weeks is as follows:

14 15 8 16 18 4 10 12 15 8

By calculating appropriate numerical measures, state, with a reason, whether or not the Poisson distribution Po(12.0) could provide a suitable model for the number of places booked each week on the database course. *(3 marks)*

3 (a) The continuous random variable T follows a rectangular distribution with probability density function given by

$$f(t) = \begin{cases} k & -a \leqslant t \leqslant b \\ 0 & \text{otherwise} \end{cases}$$

(i) Express k in terms of a and b. *(1 mark)*

(ii) Prove, using integration, that $E(T) = \frac{1}{2}(b - a)$. *(4 marks)*

(b) The error, in minutes, made by a commuter when estimating the journey time by train into London may be modelled by the random variable T with probability density function

$$f(t) = \begin{cases} \frac{1}{10} & -4 \leqslant t \leqslant 6 \\ 0 & \text{otherwise} \end{cases}$$

(i) Write down the value of $E(T)$. *(1 mark)*

(ii) Calculate $P(T < -3 \text{ or } T > 3)$. *(2 marks)*

4 A speed camera was used to measure the speed, V mph, of John's serves during a tennis singles championship.

For 10 randomly selected serves,

$$\sum v = 1179 \qquad \text{and} \qquad \sum (v - \bar{v})^2 = 1014.9$$

where $\bar{v}$ is the sample mean.

(a) Construct a 99% confidence interval for the mean speed of John's serves at this tennis championship, stating any assumption that you make. *(7 marks)*

(b) Hence comment on John's claim that, at this championship, he consistently served at speeds in excess of 130 mph. *(1 mark)*

5 A discrete random variable X has the probability distribution

$$P(X = x) = \begin{cases} \dfrac{x}{20} & x = 1, 2, 3, 4, 5 \\ \dfrac{x}{24} & x = 6 \\ 0 & \text{otherwise} \end{cases}$$

(a) Calculate $P(X \geqslant 5)$. *(2 marks)*

(b) (i) Show that $E\left(\dfrac{1}{X}\right) = \dfrac{7}{24}$. *(2 marks)*

(ii) Hence, or otherwise, show that $\text{Var}\left(\dfrac{1}{X}\right) = 0.036$, correct to three decimal places. *(3 marks)*

(c) Calculate the mean and the variance of A, the area of rectangles having sides of length $X + 3$ and $\dfrac{1}{X}$. *(5 marks)*

6 A survey is carried out in an attempt to determine whether the salary achieved by the age of 30 is associated with having had a university education.

The results of this survey are given in the table.

	Salary < £30 000	**Salary $\geqslant$ £30 000**	**Total**
University education	52	78	130
No university education	63	57	120
Total	115	135	250

(a) Use a χ^2 test, at the 10% level of significance, to determine whether the salary achieved by the age of 30 is associated with having had a university education. *(9 marks)*

(b) What do you understand by a Type I error in this context? *(2 marks)*

7 The waiting time, X minutes, for fans to gain entrance to see an event may be modelled by a continuous random variable having the distribution function defined by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2}x & 0 \leqslant x \leqslant 1 \\ \frac{1}{54}(x^3 - 12x^2 + 48x - 10) & 1 \leqslant x \leqslant 4 \\ 1 & x > 4 \end{cases}$$

(a) (i) Sketch the graph of F. *(4 marks)*

(ii) Explain why the value of q_1, the lower quartile of X, is $\frac{1}{2}$. *(2 marks)*

(iii) Show that the upper quartile, q_3, satisfies $1.6 < q_3 < 1.7$. *(3 marks)*

(b) The probability density function of X is defined by

$$\mathrm{f}(x) = \begin{cases} \alpha & 0 \leqslant x \leqslant 1 \\ \beta(x-4)^2 & 1 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(i) Show that the **exact** values of α and β are $\frac{1}{2}$ and $\frac{1}{18}$ respectively. *(5 marks)*

(ii) Hence calculate $\mathrm{E}(X)$. *(5 marks)*

END OF QUESTIONS

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