

# **General Certificate of Education**

# **Mathematics 6360**

# MFP4 Further Pure 4

# **Mark Scheme**

2008 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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М	mark is for method					
m or dM	mark is dependent on one or more M marks and is for method					
А	mark is dependent on M or m marks and is for accuracy					
В	mark is independent of M or m marks and is for method and accuracy					
Е	mark is for explanation					
$\sqrt{0}$ or ft or F	follow through from previous					
	incorrect result	MC	mis-copy			
CAO	correct answer only	MR	mis-read			
CSO	correct solution only	RA	required accuracy			
AWFW	anything which falls within	FW	further work			
AWRT	anything which rounds to	ISW	ignore subsequent work			
ACF	any correct form	FIW	from incorrect work			
AG	answer given	BOD	given benefit of doubt			
SC	special case	WR	work replaced by candidate			
OE	or equivalent	FB	formulae book			
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme			
-x EE	deduct x marks for each error	G	graph			
NMS	no method shown	с	candidate			
PI	possibly implied	sf	significant figure(s)			
SCA	substantially correct approach	dp	decimal place(s)			

## Key to mark scheme and abbreviations used in marking

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

## Otherwise we require evidence of a correct method for any marks to be awarded.

0	Solution	Marks	Total	Comments
1(a)	Rotation	M1		
	about the y-axis	A1		Ignore direction
	through $\cos^{-1}0.8$	A1	3	or $\sin^{-1} 0.6$ or $36.87^{\circ}$ or $0.644^{\circ}$
(b)	Reflection in $y = x$	M1A1	2	Ignore if it is called a line
	Total		5	
2(a)(i)	$\mathbf{a} \cdot \mathbf{b} = 0$	B1	1	
(ii)	$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & 1 \\ 1 & 1 & -5 \end{vmatrix} = \begin{bmatrix} -16 \\ 11 \\ -1 \end{bmatrix}$	M1 A1	2	
(iii)	$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 1 & -5 \\ 1 & 4 & 28 \end{vmatrix} = 0$	M1 A1	2	or via $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$ ft in this case Do not allow = 0 via (a)(i)
(b)(i)	<b>a</b> , <b>b</b> , $\mathbf{a} \times \mathbf{b}$ mutually perpendicular	B1	1	
( <b>ii</b> )	<b>a</b> , <b>b</b> , <b>c</b> co-planar	B1	1	
	Total		7	
<b>3</b> (a)	Area invariant	M1		MUST mention area
	$\Rightarrow$ Determinant = 1 $\Rightarrow pr + q^2 = 1$	A1	2	Given answer justified
(b)(i)	$\begin{bmatrix} 4 & q \\ -q & r \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $\Rightarrow 2q - 4 = 2 \text{ and } q + 2r = -1$ $\Rightarrow q = 3 \text{ and } r = -2$	M1 A1 A1	3	Either correct
(ii)	x' = 4x + 3y and $y' = -3x - 2ySetting x' = x, y' = yy = -x$	B1 M1 A1	3	
	Alternative for (b)(ii):			
	Setting $\lambda = 1$	(M2)		
	$\Rightarrow$ 3x + 3y = 0 (etc) ie y = -x	(A1)	(3)	
	Total		8	

MFP4

MFP4 (cont)						
Q	Solution	Marks	Total	Comments		
4(a)	$\mathbf{D} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}, \ \mathbf{U} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix},$	B1B1				
	$\mathbf{U}^{-1} = \begin{bmatrix} 3 & -2\\ -1 & 1 \end{bmatrix}$	B1	3	ft $\mathbf{U}^{-1}$		
(b)	$\mathbf{T}^{n} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 2^{n} & 0 \\ 0 & 2^{n} \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$	B1 M1		For $\mathbf{D}^n$ with <i>n</i> even For use of $\mathbf{U}^{-1} \mathbf{D}^n \mathbf{U}$ form		
	$= \begin{bmatrix} 2^n & 2 \times 2^n \\ 2^n & 3 \times 2^n \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$	m1 A1				
	or $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 3 \times 2^n & -2 \times 2^n \\ -2^n & 2^n \end{bmatrix}$					
	$=2^{n}\begin{bmatrix}1&0\\0&1\end{bmatrix}$	A1	5	Shown legitimately		
	Alternative for (b):					
	$\mathbf{D}^n = \begin{bmatrix} 2^n & 0\\ 0 & 2^n \end{bmatrix}$	(B1)		For $\mathbf{D}^n$ with <i>n</i> even		
	$\mathbf{T}^n = \mathbf{U} \left( 2^n \mathbf{I} \right) \mathbf{U}^{-1}$	(M1)				
	$= 2^n \left( \mathbf{U} \mathbf{I} \mathbf{U}^{-1} \right)$	(m2)				
	$=2^{n}\mathbf{I}$	(A1)	(5)	Allow $\equiv$ forms such as $3 \cdot 2^n - 2^{n+1}$		
	Total		8			

eg $3 \times (1) - (2) \implies 13y + 13z = -13$ $(3) - (2) \implies 15y + 11z = -5$ $x = 6, y = 1\frac{1}{2}, z = -2\frac{1}{2}$ Alt I (Cramer's Rule): $\begin{vmatrix} 1 & 3 & 5 \end{vmatrix} \qquad \begin{vmatrix} -2 & 3 & 5 \end{vmatrix}$	M1 A1A1 M1 A1	5	Eliminating first variable Solving $2 \times 2$ system
$x = 6$ , $y = 1\frac{1}{2}$ , $z = -2\frac{1}{2}$ Alt I (Cramer's Rule):	M1	5	Solving 2 × 2 system
Alt I (Cramer's Rule):		5	Solving $2 \times 2$ system
Alt I (Cramer's Rule):	A1	5	JOIVING 2 A 2 SYSICIN
		5	
$\begin{vmatrix} 1 & 3 & 5 \end{vmatrix}  \begin{vmatrix} -2 & 3 & 5 \end{vmatrix}$			
$\Delta = \begin{vmatrix} 1 & 3 & 5 \\ 3 & -4 & 2 \\ 3 & 11 & 13 \end{vmatrix},  \Delta_x = \begin{vmatrix} -2 & 3 & 5 \\ 7 & -4 & 2 \\ 2 & 11 & 13 \end{vmatrix},$			
$\Delta = \begin{bmatrix} 5 & -4 & 2 \end{bmatrix},  \Delta_x = \begin{bmatrix} 7 & -4 & 2 \end{bmatrix},$			
3 11 13 2 11 13			
1 - 2 - 5  $ 1 - 3 - 2 $	(M1)		Attempt at any two
$\Delta_{y} = \begin{vmatrix} 1 & -2 & 5 \\ 3 & 7 & 2 \\ 3 & 2 & 13 \end{vmatrix}, \ \Delta_{z} = \begin{vmatrix} 1 & 3 & -2 \\ 3 & -4 & 7 \\ 3 & 11 & 2 \end{vmatrix}$			
$\Delta_y = \begin{bmatrix} 3 & 7 & 2 \end{bmatrix}, \ \Delta_z = \begin{bmatrix} 3 & -4 & 7 \end{bmatrix}$			
3 2 13 3 11 2			
	( 1 1		
= 52, 312, 78 and – 130 respectively	(A1		$\Delta$ correct; $\geq 1$ other determinant correct
	A1)		
$x = \frac{\Delta_x}{\Lambda}, \ y = \frac{\Delta_y}{\Lambda}, \ z = \frac{\Delta_z}{\Lambda}$	(M1)		At least one attempted numerically
$\Delta $ , $\gamma = \Delta$ , $z = \Delta$	(1/11)		
$x = 6$ , $y = 1\frac{1}{2}$ , $z = -2\frac{1}{2}$	(A1)	(5)	
Alt II (Augmented matrix method):			
$\begin{bmatrix} 1 & 3 & 5 &   -2 \end{bmatrix}$			
$\begin{bmatrix} 1 & 3 & 5 &   & -2 \\ 3 & -4 & 2 &   & 7 \\ 3 & 11 & 13 &   & 2 \end{bmatrix} \rightarrow$	(M1)		
	(1411)		
$\begin{bmatrix} 3 & 11 & 13 \end{bmatrix} \begin{bmatrix} 2 \end{bmatrix}$			
$\begin{bmatrix} 1 & 3 & 5 &   -2 \end{bmatrix}$			
$\begin{bmatrix} 1 & 3 & 5 &   & -2 \\ 0 & -13 & -13 &   & 13 \\ 0 & 2 & -2 &   & 8 \end{bmatrix}$	(A 1)		
0 -13 -13 13	(A1)		$R_2 \rightarrow R_2 - 3R_1$
$\begin{bmatrix} 0 & 2 & -2 &   & 8 \end{bmatrix}$			$R_3 \rightarrow R_3 - 3R_1$
$\begin{bmatrix} 1 & 3 & 5 &   -2 \end{bmatrix}$			
$ \rightarrow \left  \begin{array}{ccccccc} 0 & 1 & 1 & -1 \\ 0 & 1 & -1 & 4 \end{array} \right  $	(A1)		
$\begin{vmatrix} 0 & 1 & -1 \end{vmatrix} = 4$			
$\rightarrow \left  \begin{array}{cccc} 0 & 1 & 1 \end{array} \right  \left  \begin{array}{ccc} -1 \end{array} \right $			
$\rightarrow \begin{bmatrix} 1 & 3 & 5 &   & -2 \\ 0 & 1 & 1 &   & -1 \\ 0 & 0 & -2 &   & 5 \end{bmatrix}$			$R_3 \rightarrow R_3 - R_2$
	(M1		
Substituting back to get $x = 6$ , $y = 1\frac{1}{2}$ , $z = -2\frac{1}{2}$	(M1 A1)	(5)	
$x = 0, y = 1/2, \zeta = -2/2$	A1)	$(\mathbf{J})$	
Alt III (Inverse matrix method):			
	(M1)		M0 if no inverse matrix is given
$C^{-1} = \frac{-}{52} - 33 - 2 = 13$	(A1		
$C^{-1} = \frac{1}{52} \begin{bmatrix} -74 & 16 & 26 \\ -33 & -2 & 13 \\ 45 & -2 & -13 \end{bmatrix}$	A1)		
$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = C^{-1} \begin{bmatrix} -2 \\ 7 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 1.5 \\ -2.5 \end{bmatrix}$			
$ y  = C^{-1}  7  =  1.5 $	(M1)	(5)	
	(A1)	(5)	

MFP4 (cont)							
Q	Solution	Marks	Total	Comments			
5(b)(i)	$\begin{vmatrix} 1 & 3 & 5 \\ 3 & -4 & 2 \\ a & 11 & 13 \end{vmatrix} = 26a - 26$	M1		Attempt at determinant; OE			
	Setting equal to zero and solving for $a = 1$	m1 A1	3				
(ii)	x + 3y + 5z = -23x - 4y + 2z = 7x + 11y + 13z = b						
	NB $y + z = -1$ (from before) (3) - (1) $\Rightarrow 8y + 8z = b + 2$ $b + 2 = -8 \Rightarrow b = -10$	B1 B1 M1A1	4	Equating; CAO			
	Alternative for (b)(ii): Substituting $x = 6$ , $y = 1\frac{1}{2}$ , $z = -2\frac{1}{2}$ into $x + 11y + 13z = b$ $\Rightarrow b = -10$	(M3) (A1)	(4)	Since, to be consistent, the 3 <sup>rd</sup> plane must contain the line of intersection of the first 2 planes, and therefore contains this point			
	Total		12				
6(a)(i)	$\mathbf{a} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$	B1	1				
(ii)	Equating for $\lambda: \frac{x-1}{3} = \frac{y-1}{2} = \frac{z-2}{6}$	M1 A1	2				
(iii)	$\sqrt{3^2 + 2^2 + 6^2} = 7$	B1					
	Direction cosines are $\frac{3}{7}, \frac{2}{7}$ and $\frac{6}{7}$	B1		ft on 7			
	These are the cosines of the angles between the line and the <i>x</i> -, <i>y</i> - and <i>z</i> -axes (respectively)	B1	3	Allow just "angles" correctly described			
	$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 3 & 2 \\ 1 & 1 & 3 \end{vmatrix} = 7\mathbf{i} - 10\mathbf{j} + \mathbf{k}$	M1A1					
	$d = \begin{bmatrix} 7\\5\\1 \end{bmatrix} \cdot \begin{bmatrix} 7\\-10\\1 \end{bmatrix} = 0$	M1 A1	4	ft <b>n</b>			
(ii)	$d = 0 \implies$ plane through / contains the origin	B1	1				
(c)	$\sin\theta/\cos\theta = \frac{\text{scalar product}}{\text{product of moduli}}$	M1		Must be $3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$ and their <b>n</b>			
	Numerator = $21 - 20 + 6 = 7$	B1		ft correct (unsimplified)			
	Denominator = $7.\sqrt{150}$	B1		ft both correct (unsimplified)			
	$\theta = 4.7^{\circ}$	A1	4	CAO			
	Total		15				
	Total		15				

(ii) $\begin{array}{ c c c c c } ie (\lambda - 2)(\lambda - 1)^2 = 0 \\ giving \lambda_1 = 1 (twice) and \lambda_2 = 2 \\ (ii) \\ \lambda = 1 \Rightarrow x - y + z = 0 (thrice) \\ Any two independent eigenvectors \\ (eg) \alpha \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \\ \begin{pmatrix} 1 \\ 1 \end{bmatrix} \\ \begin{pmatrix} 2 \\ 2 \end{pmatrix} - y + z = 0 \\ x - 2y + z = 0 \\ x - y = 0 \\ \end{pmatrix} $ A1 A1 A1 A1 A1 A1 A1 A1 A1 A1 A1 A1 A1 A	MFP4 (o	cont)			
$ \begin{vmatrix} \mathbf{i} & \mathbf{i} & \mathbf{j} & \mathbf{j} & \mathbf{j} & \mathbf{j} & \mathbf{j} \\ \mathbf{M}^{2} + 2\mathbf{I} = \begin{bmatrix} 4 & -3 & 3 \\ 3 & -2 & 3 \\ 3 & -3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \\ = \begin{bmatrix} 6 & -3 & 3 \\ 3 & 0 & 3 \\ 3 & -3 & 6 \end{bmatrix} = 3\mathbf{M} $ A1 3 ie $k = 3$ (ii) Multiplying by $\mathbf{M}^{-1}$ M1 A1 3 ie $k = 3$ (iii) Multiplying by $\mathbf{M}^{-1}$ M1 A1 3 ie $k = 3$ (iv) Char. eqn. is $\lambda^{3} - 4\lambda^{2}$ M1A1 A1 3 ie $(\lambda - 2)(\lambda - 1)^{2} = 0$ A1 6 ie $(\lambda - 2)(\lambda - 1)^{2} = 0$ (thrice) A1A1 A1 A	Q	Solution	Marks	Total	Comments
$ \begin{array}{ c c c c c } \mathbf{u} & \mathbf{u} $	7(a)(i)		M1		
(ii) Multiplying by $\mathbf{M}^{-1}$ to get $\mathbf{M} + 2\mathbf{M}^{-1} = 3\mathbf{I}$ so that $\mathbf{M}^{-1} = \frac{3}{2}\mathbf{I} - \frac{1}{2}\mathbf{M}$ (b)(i) Char. eqn. is $\lambda^3 - 4\lambda^2$ $+ 5\lambda - 2 = 0$ ie $(\lambda - 2)(\lambda - 1)^2 = 0$ giving $\lambda_1 = 1$ (twice) and $\lambda_2 = 2$ (ii) $\lambda = 1 \Rightarrow x - y + z = 0$ (thrice) Any two independent eigenvectors $(eg) \alpha \begin{bmatrix} 1\\1\\0 \end{bmatrix} + \beta \begin{bmatrix} 0\\1\\1\\1 \end{bmatrix}$ $\lambda = 2 \Rightarrow -y + z = 0$ x - 2y + z = 0 x - 2y + z = 0 x - y + z = 0 $(x - 2)(x - 1)^2 = 0$ M1 $\lambda = 2 \Rightarrow -y + z = 0$ x - 2y + z = 0 x - 2y + z = 0 $(x - 2y + z = 0 \Rightarrow x = y = z$ M1 $\lambda = 1 \Rightarrow x - y + z = 0$ $(x - 2y + z = 0 \Rightarrow x = y = z$ $(x - 2y + z = 0 \Rightarrow x = y = z$ M1 $\lambda = 1 \Rightarrow x - y + z = 0$ $(x - 2y + z = 0 \Rightarrow x = y = z$ M1 $\lambda = 1 \Rightarrow x - y + z = 0$ $(x - 2y + z = 0 \Rightarrow x = y = z$ M1 $\lambda = 1 \Rightarrow x - y + z = 0$ $(x - 2y + z = 0 \Rightarrow x = y = z$ M1 $\lambda = 1 \Rightarrow x - y + z = 0$ $(x - 2y + z = 0 \Rightarrow x = y = z$ M1 A1 5 Plane		$\mathbf{M}^{2} + 2\mathbf{I} = \begin{bmatrix} 4 & -3 & 3 \\ 3 & -2 & 3 \\ 3 & -3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$		2	
to get $\mathbf{M} + 2\mathbf{M}^{-1} = 3\mathbf{I}$ so that $\mathbf{M}^{-1} = \frac{3}{2}\mathbf{I} - \frac{1}{2}\mathbf{M}$ (b)(i) Char. eqn. is $\lambda^3 - 4\lambda^2$ $+ 5\lambda - 2 = 0$ ie $(\lambda - 2)(\lambda - 1)^2 = 0$ giving $\lambda_1 = 1$ (twice) and $\lambda_2 = 2$ (ii) $\lambda = 1 \Rightarrow x - y + z = 0$ (thrice) Any two independent eigenvectors $(eg) \alpha \begin{bmatrix} 1\\1\\0\\1\\1\end{bmatrix} + \beta \begin{bmatrix} 0\\1\\1\\1\\1\end{bmatrix}$ $\lambda = 2 \Rightarrow -y + z = 0$ $x - 2y + z = 0 \Rightarrow x = y = z$ M1 $\lambda = 2 \Rightarrow -y + z = 0$ $x - 2y + z = 0 \Rightarrow x = y = z$ (iii) For $\lambda = 1$ , eigenvectors represent a plane M1 f $(iii) For \lambda = 1, eigenvectors represent a planeM1f(b)(i) Char. eqn. is \lambda^3 - 4\lambda^2M1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1A1$			AI	3	1e $k = 3$
(b)(i) Char. eqn. is $\lambda^3 - 4\lambda^2$ $+ 5\lambda - 2 = 0$ giving $\lambda_1 = 1$ (twice) and $\lambda_2 = 2$ (ii) $\lambda = 1 \Rightarrow x - y + z = 0$ (thrice) Any two independent eigenvectors $(eg) \alpha \begin{bmatrix} 1\\1\\0 \end{bmatrix} + \beta \begin{bmatrix} 0\\1\\1 \end{bmatrix}$ $\lambda = 2 \Rightarrow -y + z = 0$ $x - 2y + z = 0 \Rightarrow x = y = z$ x - y = 0 (iii) For $\lambda = 1$ , eigenvectors represent a plane (b)(i) Char. eqn. is $\lambda^3 - 4\lambda^2$ M1A1 A1 A1 A1 A1 A1 A1 A1 A1 A1 A1 A1 A1 A1 A1 A1 A1 A1 A1 A1 B1 A1 A1 A1 A1 A1 A1 B1 A1 A1 A1 A1 B1 A1 A1 A1 B1 A1 A1 A1 B1 A1 A1 A1 B1 A1 A1 A1 B1 A1 A1 B1 A1 A1 A1 B1 A1 A1 A1 B1 A1 A1 B1 A1 B1 A1 A1 B1 A1 B1 A1 B1 A1 B1 A1 B1 A1 B1 A1 B1 A1 B1 A1 B1 A1 B1 A1 B1 B1 A1 B1 B1 A1 B1 B1 A1 B1 B1 A1 B1 B1 A1 B1 B1 A1 B1 B1 B1 A1 B1 B1 B1 B1 B1 A1 B1 B1 B1 B1 B1 A1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B	(ii)	to get $\mathbf{M} + 2\mathbf{M}^{-1} = 3\mathbf{I}$			
$ \begin{array}{ c c c c c } & & & & +5\lambda-2=0 & & A1A1 & & & coefficien \\ \hline ie & (\lambda-2)(\lambda-1)^2=0 & & & M1 & \\ giving & \lambda_1=1 & (twice) & and & \lambda_2=2 & & A1 & & 6 \\ \hline (ii) & & \lambda=1 \Rightarrow x-y+z=0 & (thrice) & & B1 & & \\ Any two independent eigenvectors & & & M1 & & \\ eg) & & & & \alpha \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} & & & A1 & & \\ \lambda=2 \Rightarrow -y+z=0 & & & x=y=z & & \\ & & x-2y+z=0 & \Rightarrow x=y=z & & M1 & & \\ & & & \chi-y=0 & & & \\ & & & \chi-y=0 & & & \\ \hline (iii) & & For & \lambda=1, eigenvectors represent a plane & & M1 & & \\ \hline \end{array} $			A1	3	ie $a = -\frac{1}{2}$ and $b = \frac{3}{2}$
Any two independent eigenvectorsM1Attempted(eg) $\alpha \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ A1A1 $\lambda = 2 \Rightarrow -y + z = 0$ $x - 2y + z = 0 \Rightarrow x = y = z$ M1 $\gamma \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ A15(iii) For $\lambda = 1$ , eigenvectors represent a planeM1	(b)(i)	$+5\lambda - 2 = 0$ ie $(\lambda - 2)(\lambda - 1)^2 = 0$	A1A1 M1	6	coefficients
$\begin{bmatrix} 0 & 1 \\ \lambda = 2 \Rightarrow -y + z = 0 \\ x - 2y + z = 0 \Rightarrow x = y = z \\ x - y = 0 \end{bmatrix}$ M1 $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ For $\lambda = 1$ , eigenvectors represent a plane M1 Plane	(ii)	Any two independent eigenvectors $\begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix}$	M1		Attempted
$\begin{vmatrix} x - 2y + z = 0 \implies x = y = z \\ x - y = 0 \end{vmatrix}$ M1 A1 (iii) For $\lambda = 1$ , eigenvectors represent a plane M1 Plane			AI		
(iii) For $\lambda = 1$ , eigenvectors represent a plane M1 Plane		$\begin{array}{c} x - 2y + z = 0  \Longrightarrow  x = y = z \\ x - y = 0 \end{array}$	M1		
			A1	5	
	(iii)	For $\lambda = 1$ , eigenvectors represent a plane of invariant points			Plane
For $\lambda = 2$ , eigenvectors represent an invariant lineB13		invariant line	B1		
Total 20					
TOTAL 75		TOTAL		75	