

General Certificate of Education
January 2008
Advanced Level Examination



MATHEMATICS
Unit Further Pure 3

MFP3

Friday 25 January 2008 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
 - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $f(x, y) = x^2 - y^2$

and $y(2) = 1$

(a) Use the Euler formula

$$y_{r+1} = y_r + hf(x_r, y_r)$$

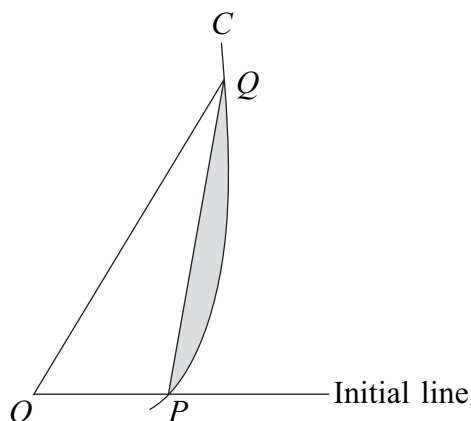
with $h = 0.1$, to obtain an approximation to $y(2.1)$. *(3 marks)*

(b) Use the formula

$$y_{r+1} = y_{r-1} + 2hf(x_r, y_r)$$

with your answer to part (a), to obtain an approximation to $y(2.2)$. *(3 marks)*

- 2 The diagram shows a sketch of part of the curve C whose polar equation is $r = 1 + \tan \theta$. The point O is the pole.



The points P and Q on the curve are given by $\theta = 0$ and $\theta = \frac{\pi}{3}$ respectively.

- (a) Show that the area of the region bounded by the curve C and the lines OP and OQ is

$$\frac{1}{2}\sqrt{3} + \ln 2 \quad (6 \text{ marks})$$

- (b) Hence find the area of the shaded region bounded by the line PQ and the arc PQ of C .
(3 marks)

- 3 (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 5 \quad (6 \text{ marks})$$

- (b) Hence express y in terms of x , given that $y = 2$ and $\frac{dy}{dx} = 3$ when $x = 0$.
(4 marks)

- 4 (a) Explain why $\int_1^{\infty} xe^{-3x} dx$ is an improper integral. (1 mark)

- (b) Find $\int xe^{-3x} dx$. (3 marks)

- (c) Hence evaluate $\int_1^{\infty} xe^{-3x} dx$, showing the limiting process used. (3 marks)

5 By using an integrating factor, find the solution of the differential equation

$$\frac{dy}{dx} + \frac{4x}{x^2 + 1} y = x$$

given that $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$. (9 marks)

6 A curve C has polar equation

$$r^2 \sin 2\theta = 8$$

(a) Find the cartesian equation of C in the form $y = f(x)$. (3 marks)

(b) Sketch the curve C . (1 mark)

(c) The line with polar equation $r = 2 \sec \theta$ intersects C at the point A . Find the polar coordinates of A . (4 marks)

7 (a) (i) Write down the expansion of $\ln(1 + 2x)$ in ascending powers of x up to and including the term in x^3 . (2 marks)

(ii) State the range of values of x for which this expansion is valid. (1 mark)

(b) (i) Given that $y = \ln \cos x$, find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$. (4 marks)

(ii) Find the value of $\frac{d^4y}{dx^4}$ when $x = 0$. (3 marks)

(iii) Hence, by using Maclaurin's theorem, show that the first two non-zero terms in the expansion, in ascending powers of x , of $\ln \cos x$ are

$$-\frac{x^2}{2} - \frac{x^4}{12} \quad (2 \text{ marks})$$

(c) Find

$$\lim_{x \rightarrow 0} \left[\frac{x \ln(1 + 2x)}{x^2 - \ln \cos x} \right] \quad (3 \text{ marks})$$

8 (a) Given that $x = e^t$ and that y is a function of x , show that:

(i) $x \frac{dy}{dx} = \frac{dy}{dt}$; *(3 marks)*

(ii) $x^2 \frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} - \frac{dy}{dt}$. *(3 marks)*

(b) Hence find the general solution of the differential equation

$$x^2 \frac{d^2y}{dx^2} - 6x \frac{dy}{dx} + 6y = 0$$
(5 marks)

END OF QUESTIONS

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