

General Certificate of Education  
January 2008  
Advanced Level Examination



**MATHEMATICS**  
**Unit Further Pure 2**

**MFP2**

Thursday 31 January 2008 9.00 am to 10.30 am

**For this paper you must have:**

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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1 (a) Express  $4 + 4i$  in the form  $re^{i\theta}$ , where  $r > 0$  and  $-\pi < \theta \leq \pi$ . (3 marks)

(b) Solve the equation

$$z^5 = 4 + 4i$$

giving your answers in the form  $re^{i\theta}$ , where  $r > 0$  and  $-\pi < \theta \leq \pi$ . (5 marks)

2 (a) Show that

$$(2r + 1)^3 - (2r - 1)^3 = 24r^2 + 2 \quad (3 \text{ marks})$$

(b) Hence, using the method of differences, show that

$$\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1) \quad (6 \text{ marks})$$

3 A circle  $C$  and a half-line  $L$  have equations

$$|z - 2\sqrt{3} - i| = 4$$

and  $\arg(z + i) = \frac{\pi}{6}$

respectively.

(a) Show that:

(i) the circle  $C$  passes through the point where  $z = -i$ ; (2 marks)

(ii) the half-line  $L$  passes through the centre of  $C$ . (3 marks)

(b) On one Argand diagram, sketch  $C$  and  $L$ . (4 marks)

(c) Shade on your sketch the set of points satisfying both

$$|z - 2\sqrt{3} - i| \leq 4$$

and  $0 \leq \arg(z + i) \leq \frac{\pi}{6}$  (2 marks)

**4** The cubic equation

$$z^3 + iz^2 + 3z - (1 + i) = 0$$

has roots  $\alpha$ ,  $\beta$  and  $\gamma$ .

(a) Write down the value of:

(i)  $\alpha + \beta + \gamma$ ; *(1 mark)*

(ii)  $\alpha\beta + \beta\gamma + \gamma\alpha$ ; *(1 mark)*

(iii)  $\alpha\beta\gamma$ . *(1 mark)*

(b) Find the value of:

(i)  $\alpha^2 + \beta^2 + \gamma^2$ ; *(3 marks)*

(ii)  $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$ ; *(4 marks)*

(iii)  $\alpha^2\beta^2\gamma^2$ . *(2 marks)*

(c) Hence write down a cubic equation whose roots are  $\alpha^2$ ,  $\beta^2$  and  $\gamma^2$ . *(2 marks)*

**5** Prove by induction that for all integers  $n \geq 1$ 

$$\sum_{r=1}^n (r^2 + 1)(r!) = n(n + 1)! \quad (7 \text{ marks})$$

**Turn over for the next question**

- 6 (a) (i) By applying De Moivre's theorem to  $(\cos \theta + i \sin \theta)^3$ , show that

$$\cos 3\theta = \cos^3 \theta - 3 \cos \theta \sin^2 \theta \quad (3 \text{ marks})$$

- (ii) Find a similar expression for  $\sin 3\theta$ . (1 mark)

- (iii) Deduce that

$$\tan 3\theta = \frac{\tan^3 \theta - 3 \tan \theta}{3 \tan^2 \theta - 1} \quad (3 \text{ marks})$$

- (b) (i) Hence show that  $\tan \frac{\pi}{12}$  is a root of the cubic equation

$$x^3 - 3x^2 - 3x + 1 = 0 \quad (3 \text{ marks})$$

- (ii) Find two other values of  $\theta$ , where  $0 < \theta < \pi$ , for which  $\tan \theta$  is a root of this cubic equation. (2 marks)

- (c) Hence show that

$$\tan \frac{\pi}{12} + \tan \frac{5\pi}{12} = 4 \quad (2 \text{ marks})$$

- 7 (a) Given that  $y = \ln \tanh \frac{x}{2}$ , where  $x > 0$ , show that

$$\frac{dy}{dx} = \operatorname{cosech} x \quad (6 \text{ marks})$$

- (b) A curve has equation  $y = \ln \tanh \frac{x}{2}$ , where  $x > 0$ . The length of the arc of the curve between the points where  $x = 1$  and  $x = 2$  is denoted by  $s$ .

- (i) Show that

$$s = \int_1^2 \coth x \, dx \quad (2 \text{ marks})$$

- (ii) Hence show that  $s = \ln(2 \cosh 1)$ . (4 marks)

**END OF QUESTIONS**