General Certificate of Education
January 2008
Advanced Subsidiary Examination

MATHEMATICS **MD01**
Unit Decision 1

Tuesday 15 January 2008 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Questions 2, 4 and 5 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MD01.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Answer **all** questions.

1 Five people, A, B, C, D and E, are to be matched to five tasks, J, K, L, M and N. The table shows the tasks that each person is able to undertake.

Person	**Task**
A	J, N
B	J, L
C	L, N
D	M, N
E	K, M

(a) Show this information on a bipartite graph. *(2 marks)*

(b) Initially, A is matched to task N, B to task J, C to task L, and E to task M.

Complete the alternating path $D-M$ ..., from this initial matching, to demonstrate how each person can be matched to a task. *(3 marks)*

2 [**Figure 1**, printed on the insert, is provided for use in this question.]

The feasible region of a linear programming problem is represented by

$$x + y \leqslant 30$$
$$2x + y \leqslant 40$$
$$y \geqslant 5$$
$$x \geqslant 4$$
$$y \geqslant \tfrac{1}{2}x$$

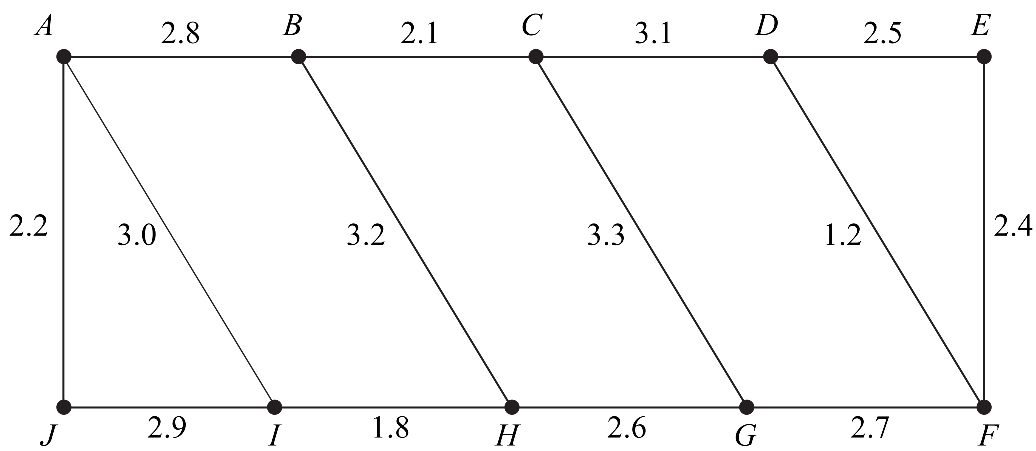
(a) On **Figure 1**, draw a suitable diagram to represent these inequalities and indicate the feasible region. *(5 marks)*

(b) Use your diagram to find the maximum value of F, on the feasible region, in the case where:

(i) $F = 3x + y$; *(2 marks)*

(ii) $F = x + 3y$. *(2 marks)*

3 The diagram shows 10 bus stops, A, B, C, ..., J, in Geneva. The number on each edge represents the distance, in kilometres, between adjacent bus stops.



The city council is to connect these bus stops to a computer system which will display waiting times for buses at each of the 10 stops. Cabling is to be laid between some of the bus stops.

(a) Use Kruskal's algorithm, showing the order in which you select the edges, to find a minimum spanning tree for the 10 bus stops. *(5 marks)*

(b) State the minimum length of cabling needed. *(1 mark)*

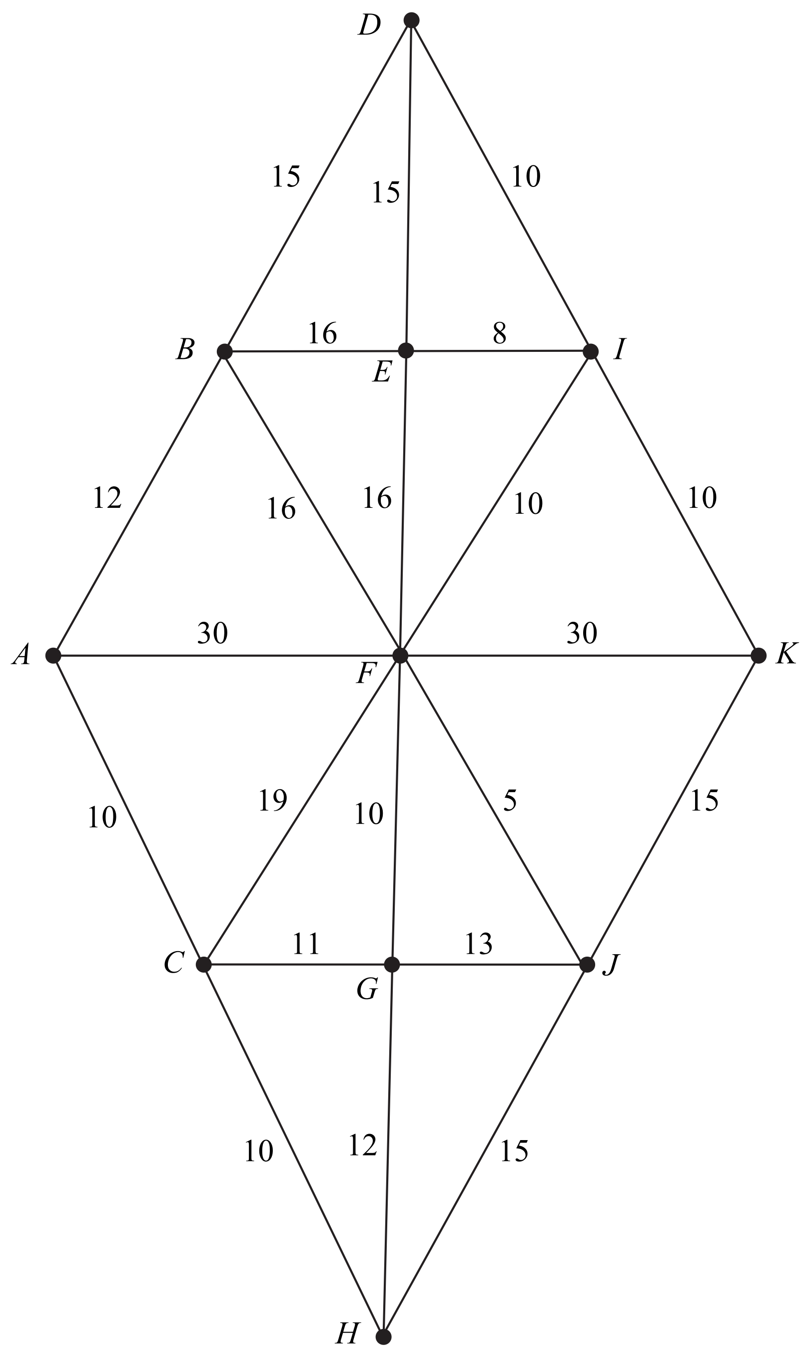
(c) Draw your minimum spanning tree. *(2 marks)*

(d) If Prim's algorithm, starting from A, had been used to find the minimum spanning tree, state which edge would have been the final edge to complete the minimum spanning tree. *(2 marks)*

Turn over for the next question

4 [**Figure 2**, printed on the insert, is provided for use in this question.]

The network shows 11 towns. The times, in minutes, to travel between pairs of towns are indicated on the edges.



The total of all of the times is 308 minutes.

(a) (i) Use Dijkstra's algorithm on **Figure 2** to find the minimum time to travel from A to K. *(6 marks)*

(ii) State the corresponding route. *(1 mark)*

(b) Find the length of an optimum Chinese postman route around the network, starting and finishing at A. (The minimum time to travel from D to H is 40 minutes.) *(5 marks)*

5 [**Figure 3**, printed on the insert, is provided for use in this question.]

(a) James is solving a travelling salesperson problem.

(i) He finds the following upper bounds: 43, 40, 43, 41, 55, 43, 43.

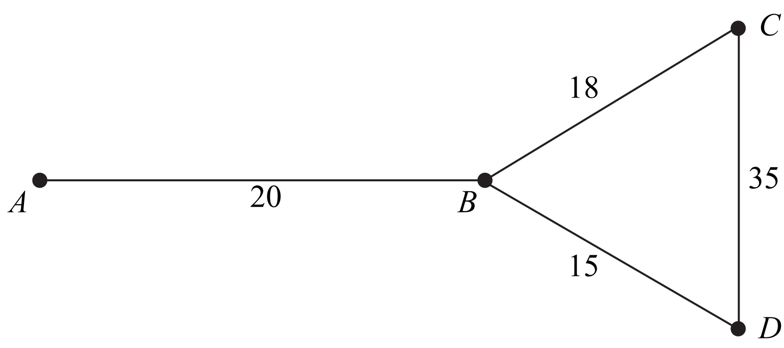
Write down the best upper bound. *(1 mark)*

(ii) James finds the following lower bounds: 33, 40, 33, 38, 33, 38, 38.

Write down the best lower bound. *(1 mark)*

(b) Karen is solving a different travelling salesperson problem and finds an upper bound of 55 and a lower bound of 45. Write down an interpretation of these results. *(1 mark)*

(c) The diagram below shows roads connecting 4 towns, *A*, *B*, *C* and *D*. The numbers on the edges represent the lengths of the roads, in kilometres, between adjacent towns.



Xiong lives at town *A* and is to visit each of the other three towns before returning to town *A*. She wishes to find a route that will minimise her travelling distance.

(i) Complete **Figure 3**, on the insert, to show the shortest distances, in kilometres, between **all** pairs of towns. *(2 marks)*

(ii) Use the nearest neighbour algorithm on **Figure 3** to find an upper bound for the minimum length of a tour of this network that starts and finishes at *A*. *(3 marks)*

(iii) Hence find the actual route that Xiong would take in order to achieve a tour of the same length as that found in part (c)(ii). *(2 marks)*

6 A student is solving cubic equations that have three different positive integer solutions.

The algorithm that the student is using is as follows:

Line 10 Input A, B, C, D
Line 20 Let $K = 1$
Line 30 Let $N = 0$
Line 40 Let $X = K$
Line 50 Let $Y = AX^3 + BX^2 + CX + D$
Line 60 If $Y \neq 0$ then go to Line 100
Line 70 Print X, "is a solution"
Line 80 Let $N = N + 1$
Line 90 If $N = 3$ then go to Line 120
Line 100 Let $K = K + 1$
Line 110 Go to Line 40
Line 120 End

(a) Trace the algorithm in the case where the input values are:

(i) $A = 1$, $B = -6$, $C = 11$ and $D = -6$; *(4 marks)*

(ii) $A = 1$, $B = -10$, $C = 29$ and $D = -20$. *(4 marks)*

(b) Explain where and why this algorithm will fail if $A = 0$. *(2 marks)*

7 The numbers 17, 3, 16 and 4 are to be sorted into ascending order.

The following four methods are to be compared: bubble sort, shuttle sort, Shell sort and quick sort (with the first number used as the pivot).

A student uses each of the four methods and produces the correct solutions below. Each solution shows the order of the numbers after each pass.

Solution 1	17	3	16	4
	3	17	16	4
	3	16	17	4
	3	4	16	17
Solution 2	17	3	16	4
	16	3	17	4
	3	4	16	17
Solution 3	17	3	16	4
	3	16	4	17
	3	16	4	17
	3	4	16	17
Solution 4	17	3	16	4
	3	16	4	17
	3	4	16	17
	3	4	16	17

(a) Write down which of the four solutions is the bubble sort, the shuttle sort, the Shell sort and the quick sort. *(3 marks)*

(b) For each of the four solutions, write down the number of comparisons and swaps (exchanges) on the first pass. *(8 marks)*

Turn over for the next question

8 Each day, a factory makes three types of hinge: basic, standard and luxury. The hinges produced need three different components: type A, type B and type C.

Basic hinges need 2 components of type A, 3 components of type B and 1 component of type C.

Standard hinges need 4 components of type A, 2 components of type B and 3 components of type C.

Luxury hinges need 3 components of type A, 4 components of type B and 5 components of type C.

Each day, there are 360 components of type A available, 270 of type B and 450 of type C.

Each day, the factory must use at least 720 components in total.

Each day, the factory must use at least 40% of the total components as type A.

Each day, the factory makes x basic hinges, y standard hinges and z luxury hinges.

In addition to $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$, find five inequalities, each involving x, y and z, which must be satisfied. Simplify each inequality where possible. *(8 marks)*

END OF QUESTIONS

Surname		Other Names	
Centre Number		Candidate Number	
Candidate Signature			

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Insert

Insert for use in **Questions 2, 4 and 5**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Figure 1 (for use in Question 2)

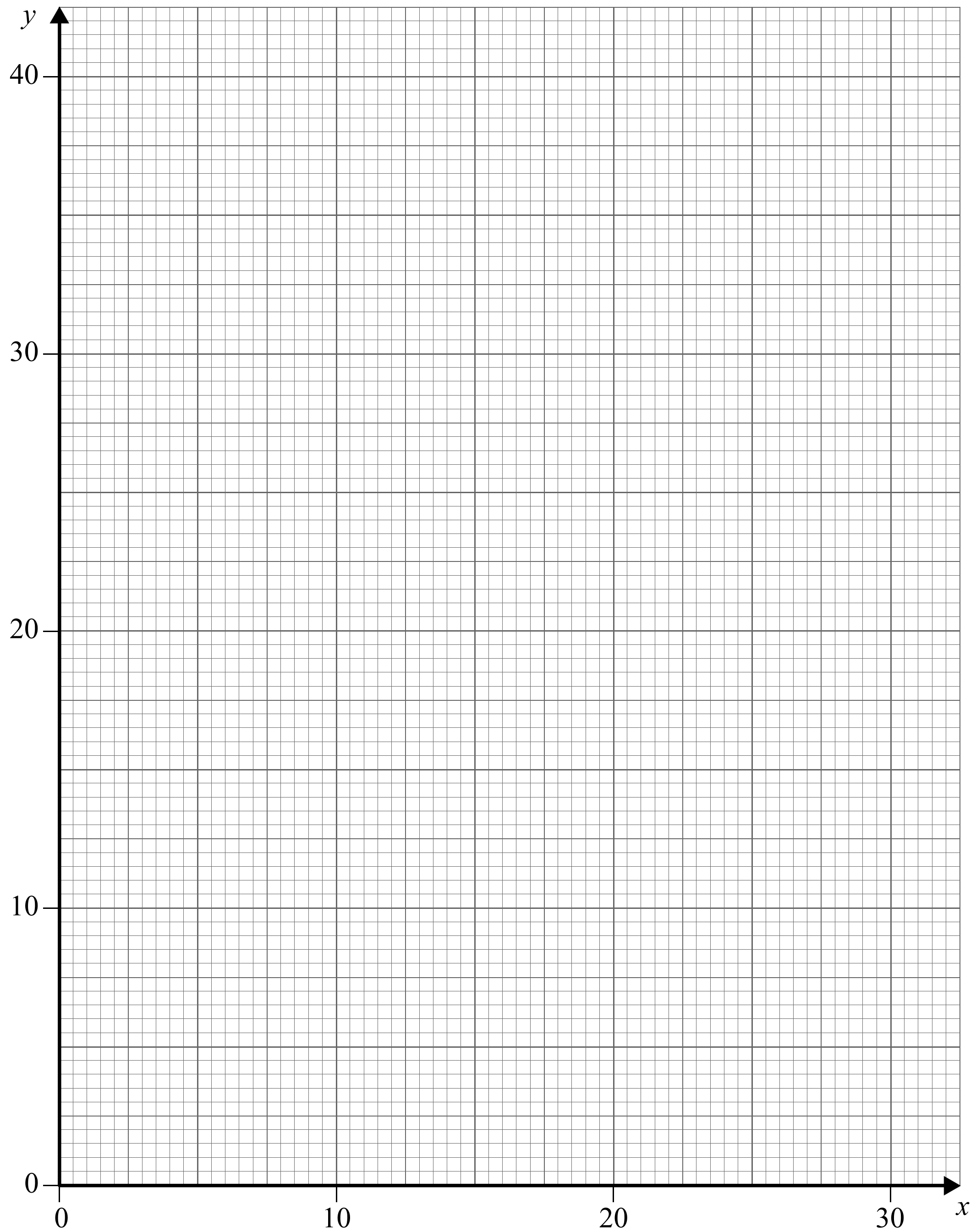

Figure 2 (for use in Question 4)

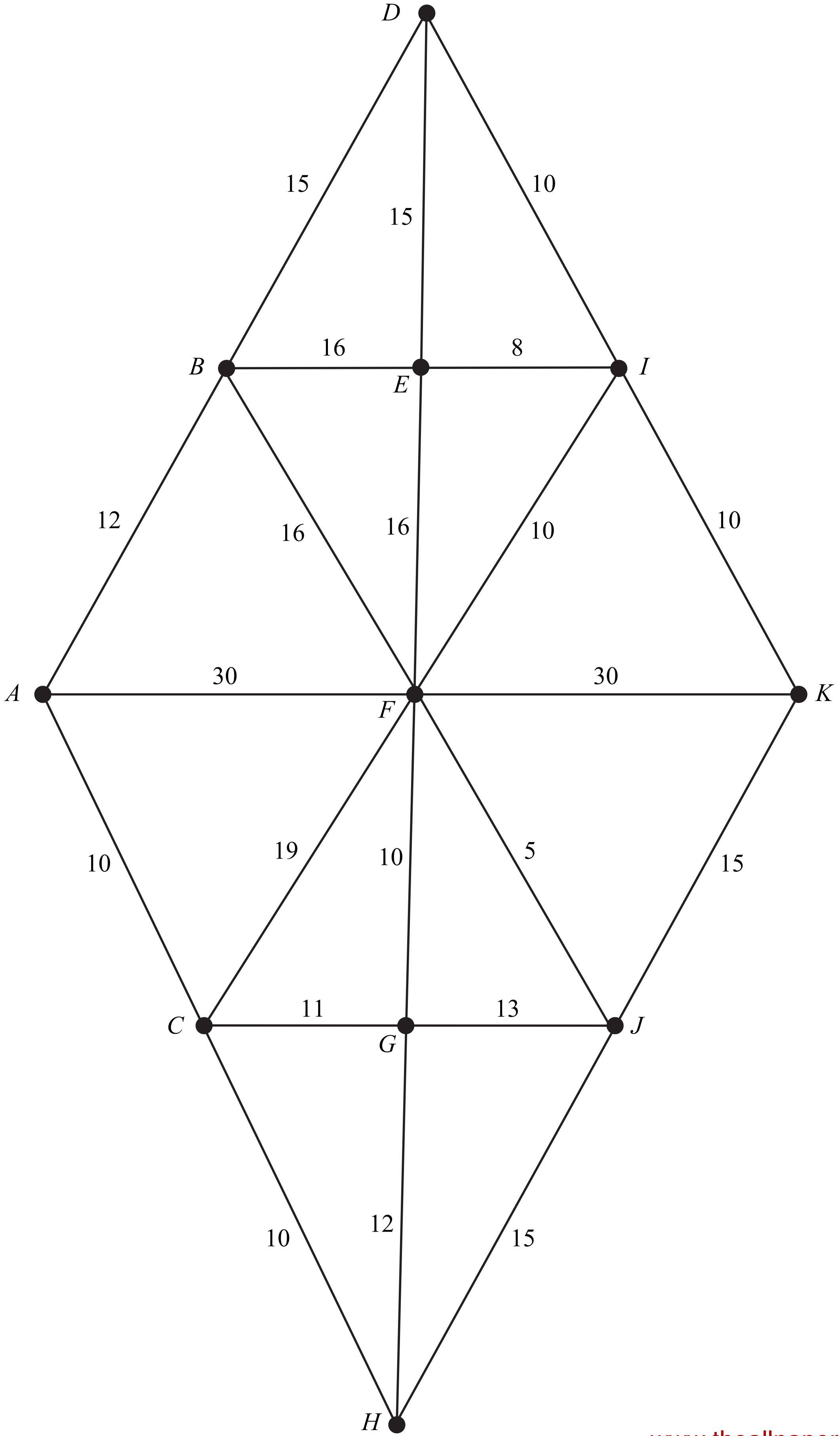

Figure 3 (for use in Question 5)

	A	*B*	*C*	*D*
A	—		38	
B		—		
C	38		—	
D				—