General Certificate of Education
January 2008
Advanced Level Examination

MATHEMATICS **MPC4**
Unit Pure Core 4

Thursday 24 January 2008 9.00 am to 10.30 am

For this paper you must have: • an 8-page answer book • the blue AQA booklet of formulae and statistical tables. You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Given that $\dfrac{3}{9-x^2}$ can be expressed in the form $k\left(\dfrac{1}{3+x}+\dfrac{1}{3-x}\right)$, find the value of the rational number k. *(2 marks)*

(b) Show that $\displaystyle\int_1^2 \frac{3}{9-x^2}\,\mathrm{d}x = \tfrac{1}{2}\ln\left(\frac{a}{b}\right)$, where a and b are integers. *(3 marks)*

2 (a) The polynomial $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 2x^3 + 3x^2 - 18x + 8$.

(i) Use the Factor Theorem to show that $(2x-1)$ is a factor of $\mathrm{f}(x)$. *(2 marks)*

(ii) Write $\mathrm{f}(x)$ in the form $(2x-1)(x^2+px+q)$, where p and q are integers. *(2 marks)*

(iii) Simplify the algebraic fraction $\dfrac{4x^2+16x}{2x^3+3x^2-18x+8}$. *(2 marks)*

(b) Express the algebraic fraction $\dfrac{2x^2}{(x+5)(x-3)}$ in the form $A + \dfrac{B+Cx}{(x+5)(x-3)}$, where A, B and C are integers. *(4 marks)*

3 (a) Obtain the binomial expansion of $(1+x)^{\frac{1}{2}}$ up to and including the term in x^2. *(2 marks)*

(b) Hence obtain the binomial expansion of $\sqrt{1+\tfrac{3}{2}x}$ up to and including the term in x^2. *(2 marks)*

(c) Hence show that $\sqrt{\dfrac{2+3x}{8}} \approx a + bx + cx^2$ for small values of x, where a, b and c are constants to be found. *(2 marks)*

4 David is researching changes in the selling price of houses. One particular house was sold on 1 January 1885 for £20. Sixty years later, on 1 January 1945, it was sold for £2000. David proposes a model

$$P = Ak^t$$

for the selling price, £P, of this house, where t is the time in years after 1 January 1885 and A and k are constants.

(a) (i) Write down the value of A. *(1 mark)*

(ii) Show that, to six decimal places, $k = 1.079775$. *(2 marks)*

(iii) Use the model, with this value of k, to estimate the selling price of this house on 1 January 2008. Give your answer to the nearest £1000. *(2 marks)*

(b) For another house, which was sold for £15 on 1 January 1885, David proposes the model

$$Q = 15 \times 1.082709^t$$

for the selling price, £Q, of this house t years after 1 January 1885. Calculate the year in which, according to these models, these two houses would have had the same selling price. *(4 marks)*

5 A curve is defined by the parametric equations $x = 2t + \frac{1}{t^2}$, $y = 2t - \frac{1}{t^2}$.

(a) At the point P on the curve, $t = \frac{1}{2}$.

(i) Find the coordinates of P. *(2 marks)*

(ii) Find an equation of the tangent to the curve at P. *(5 marks)*

(b) Show that the cartesian equation of the curve can be written as

$$(x - y)(x + y)^2 = k$$

where k is an integer. *(3 marks)*

Turn over for the next question

6 A curve has equation $3xy - 2y^2 = 4$.

Find the gradient of the curve at the point $(2, 1)$. *(5 marks)*

7 (a) (i) Express $6\sin\theta + 8\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give your value for α to the nearest $0.1°$. *(2 marks)*

(ii) Hence solve the equation $6\sin 2x + 8\cos 2x = 7$, giving all solutions to the nearest $0.1°$ in the interval $0° < x < 360°$. *(4 marks)*

(b) (i) Prove the identity $\dfrac{\sin 2x}{1 - \cos 2x} = \dfrac{1}{\tan x}$. *(4 marks)*

(ii) Hence solve the equation

$$\frac{\sin 2x}{1 - \cos 2x} = \tan x$$

giving all solutions in the interval $0° < x < 360°$. *(4 marks)*

8 Solve the differential equation

$$\frac{dy}{dx} = \frac{3\cos 3x}{y}$$

given that $y = 2$ when $x = \dfrac{\pi}{2}$. Give your answer in the form $y^2 = f(x)$. *(5 marks)*

9 The points A and B lie on the line l_1 and have coordinates $(2, 5, 1)$ and $(4, 1, -2)$ respectively.

(a) (i) Find the vector $\overrightarrow{AB}$. *(2 marks)*

(ii) Find a vector equation of the line l_1, with parameter λ. *(1 mark)*

(b) The line l_2 has equation $\mathbf{r} = \begin{bmatrix} 1 \\ -3 \\ -1 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$.

(i) Show that the point $P(-2, -3, 5)$ lies on l_2. *(2 marks)*

(ii) The point Q lies on l_1 and is such that PQ is perpendicular to l_2. Find the coordinates of Q. *(6 marks)*

END OF QUESTIONS