Version 1.0: 0108

ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MPC4 Pure Core 4

Mark Scheme

2008 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–x EE	deduct x marks for each error	G	graph
NMS	no method shown	C	candidate
PI	possibly implied	Sf	significant figure(s)
SCA	substantially correct approach	Dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC4

Q	Solution	Marks	Total	Comments
1(a)	$3 = k\left(3 + x + 3 - x\right)$	M1		OE $\frac{A}{3-x} + \frac{B}{3+x} \Rightarrow 6A = 3 \;\; 6B = 3$
	$k = \frac{1}{2}$	A1	2	or eg put $x = 0$, $\frac{3}{9} = k\left(\frac{1}{3} + \frac{1}{3}\right) \Rightarrow k = \frac{1}{2}$
(b)	$\int_1^2 \frac{3}{9-x^2}\,\mathrm{d}x = -\frac{1}{2}\ln\left(3-x\right) + \frac{1}{2}\ln\left(3+x\right)$	M1 A1F		$a\ln(3 \pm x)$ ft on k
	$= \frac{1}{2}\left(\left(\ln 5 - \ln 1\right) - \left(\ln 4 - \ln 2\right)\right) = \frac{1}{2}\ln\left(\frac{5}{2}\right)$	A1F	3	accept $\ln\left(\frac{10}{4}\right)$
				ft only for sign error in integral: $\frac{1}{2}\ln\left(\frac{5}{8}\right)$
	Total		**5**	

Q	Solution	Marks	Total	Comments
2(a)(i)	$\mathrm{f}\left(\frac{1}{2}\right)=2\times\left(\frac{1}{2}\right)^3+3\times\left(\frac{1}{2}\right)^2-18\left(\frac{1}{2}\right)+8$	M1		use of $\pm\frac{1}{2}$ substituted in f (x)
	$=\frac{1}{4}+\frac{3}{4}-9+8=0\Rightarrow$ factor	A1	2	arithmetic seen and conclusion – minimum seen: $2\times\frac{1}{8}+3\times\frac{1}{4}-18\times\frac{1}{2}+8=0$
(ii)	$\mathrm{f}(x)=(2x-1)(x^2+2x-8)$	B1B1	2	or $p = 2$, $q = -8$
(iii)	$\frac{4x(x+4)}{(2x-1)(x+4)(x-2)}$	M1		numerator correct; attempt to factorise denominator (algebraic fraction not required)
	$=\frac{4x}{(2x-1)(x-2)}$	A1	2	CAO
(b)	$2x^2=A(x+5)(x-3)+B+Cx$	M1		any equivalent method using PFs (see alternative method)
	$A=2$	B1		
	$2A+C=0 \qquad -15A+B=0$	M1		equate coefficients or use 2 values of x to find B and C
	$C=-4 \qquad B=30$	A1	4	both B and C correct
	ALTERNATIVE METHOD 1			
	$\begin{array}{r} 2 \\ x^2+2x-15\overline{)\,2x^2\qquad\quad} \\ \underline{2x^2+4x-30} \\ -4x+30 \end{array}$	(M1)		complete division
	$A = 2$	(B1)		
	$B = 30$	(A1)		
	$C = -4$	(A1)		
	ALTERNATIVE METHOD 2			
	$\frac{2x^2}{(x+5)(x-3)}=A+\frac{D}{x+5}+\frac{E}{x-3}$			
	$2x^2=A(x+5)(x-3)+D(x-3)+E(x+5)$			
	$x=3 \qquad 18=8E \qquad E=\frac{9}{4}$ $x=-5 \qquad 50=-8D \qquad D=-\frac{25}{4}$	(M1)		find D and E
	$x=0,\ 0=-15A+\left(-\frac{25}{4}\right)(-3)+\left(\frac{9}{4}\right)(5)$			
	A=2	(B1)		
	$\frac{D}{x+5}+\frac{E}{x-3}=\frac{-25}{4(x+5)}+\frac{9}{4(x-3)}$ $=\frac{-25(x-3)+9(x+5)}{4(x+5)(x-3)}$			
	$=\frac{120-16x}{4(x+5)(x-3)}$	(M1)		recombine to required form
	$=\frac{30-4x}{(x+5)(x-3)}$	(A1)		CAO
	Total		**10**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$(1+x)^{\frac{1}{2}}=1+\frac{1}{2}x+kx^2$	M1		
	$=1+\frac{1}{2}x-\frac{1}{8}x^2$	A1	2	
(b)	$\left(1+\frac{3}{2}x\right)^{\frac{1}{2}}=1+\frac{1}{2}\left(\frac{3}{2}x\right)-\frac{1}{8}\left(\frac{3}{2}x\right)^2$	M1		x replaced by $\frac{3}{2}x$ – condone missing brackets, but not incorrectly placed brackets eg $\left(\frac{3}{2}\right)x^2$ alternatively, start again and find correct expression
	$=1+\frac{3}{4}x-\frac{9}{32}x^2$	A1	2	correct evaluation
(c)	$\sqrt{\frac{2+3x}{8}}=\sqrt{\frac{2+3x}{4\times 2}}=k\left(1+\frac{3}{2}x\right)^{\frac{1}{2}}$	M1		manipulation to $k\times(\text{answer to (b)})$ and evaluated $\Rightarrow a+bx+cx^2$
	$=\frac{1}{2}+\frac{3}{8}x-\frac{9}{64}x^2$	A1	2	a, b, c fractions or decimals only Or use $(a+x)^n$ formula (condone one error for M1)
	Total		**6**	
4(a)(i)	$A=20$	B1	1	
(ii)	$\frac{2000}{A}=k^{60}$	M1		
	$k=(100)^{\frac{1}{60}}=1.079775$	A1	2	AG; or $k=10^{\frac{\log 100}{60}}=10^{0.0333}$ or $\sqrt[60]{100}$ or $\sqrt[30]{10}$ or $e^{\frac{\ln 100}{60}}=e^{0.076}$ or $e^{0.077}$ or 1.0797751(6) seen
(iii)	$P=20\times k^{2008-1885}$	M1		
	$=251780\approx 252000$	A1	2	CAO nearest 1000
(b)	$15\times 1.082709^t=20\times 1.079775^t$	M1		equate prices
	$\frac{15}{20}=\left(\frac{1.079775}{1.082709}\right)^t$	M1		t as a single index, or correct log expression at this stage
	$t=\frac{\log 0.75}{\log 0.997290}$	m1		expression for t
	$t=106.017\Rightarrow 1991$	A1	4	**SC** Answer only/Trial and error 106 seen (2 out of 4) 1991 (4 out of 4)
	Total		**9**	

Q	Solution	Marks	Total	Comments
5(a)(i)	$t=\frac{1}{2}$ $\quad x=2\times\frac{1}{2}+\frac{1}{\left(\frac{1}{2}\right)^2}$ $\quad y=2\times\frac{1}{2}-\frac{1}{\left(\frac{1}{2}\right)^2}$	M1		
	$x=5$ $\quad y=-3$	A1	2	
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}t}=2+2t^{-3}$ $\quad \frac{\mathrm{d}x}{\mathrm{d}t}=2-2t^{-3}$	M1A1		2 and $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{t^2}\right)$ attempted in both derivatives
	$t=\frac{1}{2}$ $\quad \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2+\frac{2}{1/8}}{2-\frac{2}{1/8}}=-\frac{9}{7}$	M1 A1		use chain rule; expressions can be in terms of t or evaluated CAO or any equivalent fraction (not decimals)
	$y+3=-\frac{9}{7}(x-5)$	B1F	5	ft on x, y and gradient if $y = mx + c$ used, c must be found correctly and the equation must be re-written
(b)	$x-y=\frac{2}{t^2}$ $\quad x+y=4t$	M1		either correct expression or both of $x-y=4t$ and $x+y=\frac{2}{t^2}$
	$\frac{2}{(x-y)}=\left(\frac{x+y}{4}\right)^2$	M1		eliminate t
	$32=(x-y)(x+y)^2$	A1	3	or $(x-y)(x+y)^2=\frac{2}{t^2}\times(4t)^2=32$ $k = 32$ alone, no marks
	Total		**10**	
6	$3x\frac{\mathrm{d}y}{\mathrm{d}x}+3y-4y\frac{\mathrm{d}y}{\mathrm{d}x}=0$	M1 A1 A1 B1		attempt implicit differentiation product chain constant
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{3}{2}$	A1	5	CSO
	ALTERNATIVE METHOD			
	$x=\frac{2}{3}y+\frac{4}{3y}$	(M1)		solve for x = expression in y and differentiate with respect to y
	$\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{2}{3}-\frac{4}{3y^2}$	(A1A1)		
	$y=1$, $\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{2}{3}-\frac{4}{3}$	(M1)		substitute $y = 1$
	$\frac{\mathrm{d}x}{\mathrm{d}y}=-\frac{3}{2}$	(A1)		CSO
	Total		**5**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$R=10$	B1		R = 10
	$\tan\alpha=\frac{8}{6},\quad \alpha=53.1$	B1F	2	For α; ft incorrect R
(ii)	$\sin\left(2x+53.1\right)=0.7$	M1		
	$2x+53.1=44.4$	A1F		one correct answer ; ft α and R
	135.6 or $135.7, 404.4, 495.6$ or 495.7	A1		3 other correct answers – ignore extras
	$x=41.2$ or 41.3, 175.6 or 175.7, 221.2 or 221.3, 355.6 or 355.7	A1	4	four solutions CAO (with decimal place discrepancies) Answers only: 0/4
(b)(i)	$\sin 2x=2\sin x\cos x$	B1		identities for $\sin 2x$ and $\cos 2x$ in any correct form
	$\cos 2x=\cos^2 x-\sin^2 x$	B1		
	$\frac{\sin 2x}{1-\cos 2x}=\frac{2\sin x\cos x}{1-\left(1-2\sin^2 x\right)}=$	M1		use of candidate's double angle formulae
	$\frac{2\sin x\cos x}{2\sin^2 x}=\frac{\cos x}{\sin x}=\frac{1}{\tan x}$	A1	4	AG, CSO
(ii)	$\frac{1}{\tan x}=\tan x \qquad \tan x=\pm 1$	M1A1		(see * below)
	x = 45,	B1		x=45
	135, 225, 315	A1	4	if answers given without working, B1 max if $\frac{1}{\tan x}$=tan x seen and followed by correct answers without working 4 out of 4
	Total		**14**	

* Comments for 7(b)(ii)

If hence ignored, so working in sines and cosines, must simplify as far as:

$\cos^2 x=\sin^2 x$	or	$\cos^2 x=\frac{1}{2}$	or	$\sin^2 x=\frac{1}{2}$	for M1
$\cos 2x=0$	or	$\cos x=\pm\frac{1}{\sqrt{2}}$	or	$\sin x=\pm\frac{1}{\sqrt{2}}$	for A1

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8	$\int y\,\mathrm{d}y = \int 3\cos 3x\,\mathrm{d}x$	M1		attempt to separate and integrate $py^2 = q\sin 3x$ seen $\Rightarrow$ implies separation
	$\frac{1}{2}y^2 = \sin 3x \quad (+C)$	A1A1		integrals – accept $\frac{1}{3}\times 3\sin 3x$
	$\left(\frac{\pi}{2}, 2\right) \quad \frac{1}{2}\times 4 = \sin\frac{3\pi}{2} + C$	M1		use $\left(\frac{\pi}{2}, 2\right)$ to find constant
	$C = 3$			
	$y^2 = 2\sin 3x + 6$	A1	5	CSO (in any correct form)
	Total		**5**	
9(a)(i)	$\overrightarrow{AB} = \begin{bmatrix}4\\1\\-2\end{bmatrix} - \begin{bmatrix}2\\5\\1\end{bmatrix} = \begin{bmatrix}2\\-4\\-3\end{bmatrix}$	M1A1	2	M1 for $\pm(\overrightarrow{OA} - \overrightarrow{OB})$
(ii)	$(\mathbf{r} =) \begin{bmatrix}2\\5\\1\end{bmatrix} + \lambda\begin{bmatrix}2\\-4\\-3\end{bmatrix}$	B1F	1	ft on $\overrightarrow{AB}$; OE
(b)(i)	$\begin{bmatrix}1\\-3\\-1\end{bmatrix} + \mu\begin{bmatrix}1\\0\\-2\end{bmatrix} = \begin{bmatrix}-2\\-3\\5\end{bmatrix}$	M1		μ found and verified or statement $\mu = -3$ satisfies all components
	$1 + \mu = -2 \qquad \mu = -3$ $-1 - 2\mu = 5 \qquad \mu = -3$	A1	2	$\mu = -3$ alone B1
	ALTERNATIVE METHOD $\mu\begin{bmatrix}1\\0\\-2\end{bmatrix} = \begin{bmatrix}-3\\0\\6\end{bmatrix}$, which is satisfied by $\mu = -3$			
(ii)	$\overrightarrow{PQ} = \left(\begin{bmatrix}2\\5\\1\end{bmatrix} + \lambda\begin{bmatrix}2\\-4\\-3\end{bmatrix}\right) - \begin{bmatrix}-2\\-3\\5\end{bmatrix} = \begin{bmatrix}4+2\lambda\\8-4\lambda\\-4-3\lambda\end{bmatrix}$	M1 A1		$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$ with $\overrightarrow{OQ}$ in parametric form in terms of λ (can be inferred later) or $\begin{bmatrix}6+2\lambda\\4-4\lambda\\-7-3\lambda\end{bmatrix}$
	$\begin{bmatrix}4+2\lambda\\8-4\lambda\\-4-3\lambda\end{bmatrix} \bullet \begin{bmatrix}1\\0\\-2\end{bmatrix}$	M1		$\overrightarrow{PQ} \bullet \begin{bmatrix}1\\0\\-2\end{bmatrix}$ with $\overrightarrow{PQ}$ in terms of λ (can be inferred later)
	$(4 + 2\lambda) + (-2)(-4 - 3\lambda) = 0$	m1		linear expression in λ equated to 0
	$\lambda = -1.5$	A1F		ft on sign/arithmetic error in $\overrightarrow{PQ}$ or equation
	Q is $(-1, 11, 5.5)$	A1	6	CAO
	Total		**11**	
	TOTAL		**75**	