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ASSESSMENT and QUALIFICATIONS ALLIANCE

## General Certificate of Education

# Mathematics 6360

## MPC2 Pure Core 2

# Mark Scheme

*2008 examination - January series*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available to download from the AQA Website: www.aqa.org.uk



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## Key to mark scheme and abbreviations used in marking

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |

| | | | |
|---|---|---|---|
| $\surd$ or ft or F | follow through from previous incorrect result | MC | mis-copy |
| CAO | correct answer only | MR | mis-read |
| CSO | correct solution only | RA | required accuracy |
| AWFW | anything which falls within | FW | further work |
| AWRT | anything which rounds to | ISW | ignore subsequent work |
| ACF | any correct form | FIW | from incorrect work |
| AG | answer given | BOD | given benefit of doubt |
| SC | special case | WR | work replaced by candidate |
| OE | or equivalent | FB | formulae book |
| A2,1 | 2 or 1 (or 0) accuracy marks | NOS | not on scheme |
| $-x$ EE | deduct $x$ marks for each error | G | graph |
| NMS | no method shown | c | candidate |
| PI | possibly implied | sf | significant figure(s) |
| SCA | substantially correct approach | dp | decimal place(s) |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**MPC2**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **1(a)** | Area of sector = $\frac{1}{2}r^2\theta$ = $\frac{1}{2}\times 6^2\times\theta$ | M1 | | $\frac{1}{2}r^2\theta$ seen or used |
| | $6 \times 3 = 2 \times \frac{1}{2}\times 6^2\times\theta$ | m1 | | OE Forming equation |
| | $36\theta = 18 \Rightarrow \theta = 0.5$ | A1 | 3 | AG |
| **(b)** | Arc = $6\theta$;<br>….. = 3 cm<br>$\Rightarrow$ Perimeter = 12 + arc = 15 cm | M1<br>A1<br>A1F | 3 | $r\theta$ seen or used<br>PI by a correct perimeter<br>Ft wrong evaluation of $6\theta$. Condone missing/wrong units throughout the question. |
| | **Total** | | **6** | |
| **2(a)** | ($d$ ) = 7 | B1 | 1 | 7 |
| **(b)** | (101st term) = $a$ + (101−1) $d$<br>…………… = 51 + 100(7) = 751 | M1<br>A1F | 2 | Ft on c's answer for $d$. NMS/rep. addn., give both marks for '751'.<br>**SC**<br>if M0, award B1 for 7$n$+44 OE |
| **(c)** | $S_n = \frac{100}{2}[751+1444]$ **or**<br>$S_n = \frac{100}{2}[2\times 751+(100-1)7]$ | M1 | | Formula for{$S_n$}with [any **3** of<br>$a$ = c's 751 (condoning '751'±$d$)<br>**or** $d$ = c's 7 **or** $n$ =100 **or**<br>$l$ = 1444 substituted] or [$S_{200}-S_k$ with $k$=100, (condoning $k$=99 or 101) stated/used with correct ft substitution in $S_{200}$ or $S_k$ ] |
| | = 109 750 | A1 | 2 | |
| | **Total** | | **5** | |
| **3(a)** | $\frac{BC}{\sin 72} = \frac{18.7}{\sin 50}$ [=24.4….] | M1 | | Use of the sine rule |
| | $BC = \frac{18.7\sin 72}{\sin 50}$ | m1 | | Rearrangement |
| | ($BC$)=23.21(6..){= 23.2 to nearest 0.1cm} | A1 | 3 | AG Need >1dp if using cm eg 23.21 or 23.22; at least 1dp if using mm. |
| **(b)** | Angle $C$ = 180° – (50°+72°) = 58° | M1 | | Valid method to find either angle $C$ (PI eg by sin $C$ =0.848(04..)) or side $AB$ |
| | Area of triangle = 0.5×18.7×23.2..×sin $C$ | M1 | | OE eg 0.5×18.7×$AB$×sin72° |
| | ….. = 184 cm$^2$ | A1 | 3 | Accept 183.8 to 184.2<br>Condone missing/wrong units |
| | **Total** | | **6** | |

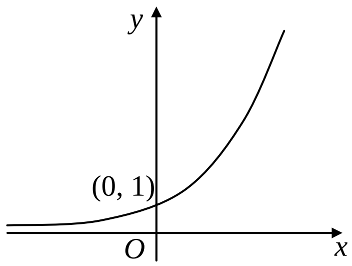
**MPC2 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 4 | $h = 1$ | B1 | | PI |
| | $\text{I} \approx \frac{h}{2}\{\ldots\}$ | M1 | | OE summing of areas of the three 'trapezia' |
| | $\{\ldots\} = \text{f}(0)+\text{f}(3)+2[\text{f}(1)+\text{f}(2)]$ <br> $\{\ldots\} = \sqrt{3} + \sqrt{12} + 2[\sqrt{4} + \sqrt{7}]$ | A1 | | ($\sum$trap=1.866.+2.3228.+3.0549) |
| | $(\text{I} \approx)\ \frac{1}{2}[5.19615. + 2\times 4.64575...]$ | | | |
| | $= \frac{1}{2}[14.4876..] = 7.2438.. = 7.244$ | A1 | 4 | CAO Must be 3dp. |
| | **Total** | | **4** | |
| 5a(i) | $\frac{dy}{dx} = 4\times\frac{1}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 2x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$ | M1 <br> A1A1 | 3 | A power decreased by 1 <br> A1 for each correct term |
| (ii) | At $P(4,0)$, $\frac{dy}{dx} = \frac{2}{\sqrt{4}} - \frac{3}{2}\times 2$ | M1 | | Attempts $\frac{dy}{dx}$ when $x = 4$ |
| | $= 1 - 3 = -2$ | A1 | 2 | AG |
| (iii) | Gradient of normal $= \frac{1}{2}$ | M1 | | Use of or stating $m\times m' = -1$ |
| | Equation of normal is $y - 0 = m[x - 4]$ | M1 | | $m$ numerical; can be awarded even if $m = -2$ |
| | $y - 0 = \frac{1}{2}(x-4) \Rightarrow \quad 2y = x - 4$ | A1 | 3 | ACF of the equation |
| (iv) | At $Q$, $x = 0$, $2y = 0 - 4$ <br> $y_Q = -2$ | M1 <br> A1F | | PI <br> Ft on a linear equation for normal provided $y_Q$ is negative and prev A1 is lost |
| | Area of triangle $OPQ = 0.5\times 4\times \lvert y_Q \rvert$ | | | |
| | $= 4$ | B1F | 3 | Ft on c's negative $y_Q$ |
| (v) | $2x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 0 \Rightarrow 2x^{-\frac{1}{2}} = \frac{3}{2}x^{\frac{1}{2}}$ | M1 | | Puts c's $\frac{dy}{dx} = 0$ and a 1st step in attempt to solve. |
| | $2 = \frac{3}{2}x$ ; $\quad \Rightarrow x = \frac{4}{3}$ | m1; <br> A1 | 3 | Valid method to $ax=b$ <br> Condone 1.3 or better |
| (b)(i) | $\int\left(4x^{\frac{1}{2}} - x^{\frac{3}{2}}\right)dx = 4\frac{x^{\frac{3}{2}}}{1.5} - \frac{x^{\frac{5}{2}}}{2.5}\ \{+c\}$ | M1 <br> A1,A1 | 3 | One power correct <br> Condone absence of "+$c$" |
| | $= \frac{8}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\ \{+c\}$ | | | |
| (ii) | Area under curve $= 4\frac{4^{\frac{3}{2}}}{1.5} - \frac{4^{\frac{5}{2}}}{2.5} - \{0\}$ | M1 | | F(4) − {F(0)} |
| | Total area = F(4) + area triangle $OPQ$ | m1 | | |
| | Total area $= \frac{128}{15} + 4 = \frac{188}{15} = 12.5\ (3\ldots)$ | A1 | 3 | Accept 3 sf if clear |
| | **Total** | | **20** | |

**MPC2 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **6(a)(i)** | $(1+x)^3 = 1+3x+3x^2+x^3$ | M1 | | Any valid method to expand $(1+x)^3$ fully |
| | | A1 | 2 | |
| **(ii)** | $(1+x)^4 = 1+4x+6x^2+4x^3+x^4$ | M1 | | Any valid method to expand $(1+x)^4$ fully |
| | | A1 | 2 | |
| **(b)(i)** | $(1+4x)^3 = 1+3(4x)+3(4x)^2+(4x)^3$ | M1 | | |
| | $(1+4x)^3 = 1+12x+48x^2+64x^3$ | A1√ | 2 | Ft on one numerical slip in (a)(i) |
| **(ii)** | $(1+3x)^4$ | | | |
| | $= 1+4(3x)+6(3x)^2+4(3x)^3+(3x)^4$ | M1 | | |
| | $= 1 + 12x + 54x^2 + 108x^3 + 81x^4$ | A1√ | 2 | Ft on one numerical slip in (a)(ii) |
| **(c)** | $(1+3x)^4 - (1+4x)^3 = 1 + 12x + 54x^2 +$ | | | |
| | $108x^3 + 81x^4 - (1+12x+48x^2+64x^3)$ | M1 | | Subtracts the answers to (b) with correct number of terms and combines at least two pairs of like terms. |
| | = 6*x*2 + 44*x*3 +81*x*4 | A1 | 2 | CAO |
| | | | | **SC**: If no attempt in (b) but full expansions given in working for (c), mark retrospectively. |
| | **Total** | | **10** | |
| **7(a)** | $x = 8$ | B1 | 1 | No clear log law errors seen. Condone answer left as $\frac{16}{2}$ |
| **(b)** | $\log_a y = \log_a 3^2 + \log_a 4 + 1$ | M1 | | One law of logs used correctly |
| | $\log_a y = \log_a (3^2 \times 4) + 1$ | M1 | | Either a second law of logs used correctly or the 1 written as $\log_a a$ |
| | $\log_a y = \log_a (3^2 \times 4) + \log_a a = \log_a 36a$ | | | |
| | $\Rightarrow y = 36a$ | A1 | 3 | CSO |
| | **Total** | | **4** | |

**MPC2 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **8(a)** |  | B1<br><br>B1 | <br><br>2 | Shape (graph must clearly go below the intersection pt.). Condone if $x$-axis is a tangent<br><br>Only intersection with $y$-axis at (0, 1) stated/indicated … (accept 1 on $y$-axis as equivalent) 0 |
| **(b)(i)** | Stretch **(I)** in $x$-direction **(II)** scale factor 0.5 **(III)** | M1<br>A1 | <br>2 | Need**(I)** & one of **(II),(III)**<br>M0 if >1 transformation |
| **(ii)** | Translation; | B1; | | Must be 'Translation' or 'translate(d)' for 1st B mark |
| | $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$ | B1 | 2 | Accept **full** equivalent to vector in words provided linked to 'translation/ move/shift' and **negative** $x$-direction (Note: B0 B1 is possible) |
| | **ALTn:** Stretch **(I)** in $y$-direction **(II)** scale factor 3 **(III)** | | | [Mark the alternative as in (b)(i).] |
| **(c)(i)** | $9^x = (3^2)^x = 3^{2x} = (3^x)^2 = Y^2$;<br>$3^{x+1} = 3^x \times 3^1 = 3Y$ | M1 | | Justifying either $9^x = Y^2$ or $3^{x+1} = 3Y$ |
| | $9^x - 3^{x+1} + 2 = 0 \Rightarrow Y^2 - 3Y + 2 = 0$<br>$\Rightarrow (Y-1)(Y-2) = 0$ | A1 | 2 | AG |
| **(ii)** | $Y = 1 \Rightarrow \quad 3^x = 1 \Rightarrow x = 0$ | B1 | | AG (Accept direct substitution if convinced) |
| | $Y = 2 \Rightarrow \quad 3^x = 2$<br>$\log_{10} 3^x = \log_{10} 2$ | M1 | | Takes logs of both, PI by 'correct' value(s) later.<br>or $x = \log_3 2$ seen |
| | $x \log_{10} 3 = \log_{10} 2$ | m1 | | Use of $\log 3^x = x \log 3$ or $\log_3 2 = \frac{\lg 2}{\lg 3}$ OE (PI by $\log_3 2$ = 0.630 or 0.631 or better) |
| | $x = \frac{\lg 2}{\lg 3} = 0.630929....$ =0.6309 to 4dp | A1 | 4 | Must show that logarithms have been used otherwise 0/3 |
| | **Total** | | **12** | |

**MPC2 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **9(a)** | $\dfrac{3+\sin^2\theta}{\cos\theta-2}=3\cos\theta$ | | | |
| | $\Rightarrow \dfrac{3+(1-\cos^2\theta)}{\cos\theta-2}=3\cos\theta$ | M1 | | $\cos^2\theta + \sin^2\theta = 1$ stated or used [If cand starts with cos $\theta = -½$ and gets $\sin^2\theta = ¾$ without explicitly finding value for $\theta$ and verifies 1st equation is true, award M1moA0] |
| | $\Rightarrow \dfrac{4-\cos^2\theta}{\cos\theta-2}=3\cos\theta$ | | | |
| | $\Rightarrow \dfrac{(2-\cos\theta)(2+\cos\theta)}{\cos\theta-2}=3\cos\theta$ | m1 | | Difference of two squares<br>or division (PI by next line) |
| | $\Rightarrow -1(2+\cos\theta)=3\cos\theta$ | A1 | | |
| | $\Rightarrow -2=4\cos\theta \Rightarrow \cos\theta=-\dfrac{1}{2}$ | A1 | 4 | CSO AG |
| | **Alternative for (a)** | | | |
| | $3+1-\cos^2\theta=3\cos^2\theta-6\cos\theta$ | (M1) | | $\cos^2\theta + \sin^2\theta = 1$ |
| | $(4\cos\theta+2)(\cos\theta-2)=0$ | (m1) | | Factorising or formula |
| | $\cos\theta - 2 \neq 0$ | (A1) | | Indicates rejection of $\cos\theta=2$ |
| | $\Rightarrow 4\cos\theta=-2 \Rightarrow \cos\theta=-\dfrac{1}{2}$ | (A1) | | AG Be convinced |
| **(b)** | $\theta = 3x \Rightarrow \cos 3x = -\dfrac{1}{2}$ | M1 | | Uses part (a) to reach either $\cos 3x = -0.5$ or $\cos 3x = 0.5$ |
| | $\cos^{-1}\left(-\dfrac{1}{2}\right) = 120°$ | m1 | | Or $\cos^{-1}(0.5) = 60°$ Condone radians here |
| | $3x = 120°, 240°, 480°, \ldots$ | | | |
| | $x = 40°, 80°, 160°$ | A2,1,0 | 4 | A1 for at least two correct.<br>If >3 solutions in the interval $0° < x < 180°$, deduct 1 mark from any A marks for each extra solution.<br>Deduct 1 mark from any A marks if answers in radians. Ignore extra values outside the given interval. |
| | **Total** | | **8** | |
| | **TOTAL** | | **75** | |