



# General Certificate of Education

## Mathematics 6360

### *MS2B Statistics 2B*

# Mark Scheme

## *2005 examination – June series*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

## Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
$\surd$ or ft or F	follow through from previous		
	incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	OE	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

### Application of Mark Scheme

#### **No method shown:**

Correct answer without working

mark as in scheme

Incorrect answer without working

zero marks unless specified otherwise

#### **More than one method / choice of solution:**

2 or more complete attempts, neither/none crossed out

mark both/all fully and award the mean mark rounded down

1 complete and 1 partial attempt, neither crossed out

award credit for the complete solution only

#### **Crossed out work**

do not mark unless it has not been replaced

**Alternative solution** using a correct or partially correct method

award method and accuracy marks as appropriate

**MS2B**

Q	Solution	Marks	Total	Comments																								
<b>1(a)</b>	$P(X = 2) = \frac{e^{-2.6} (2.6)^2}{2!}$ $= 0.251$	M1 A1	2	0.5184 – 0.2674 = 0.251																								
<b>(b)(i)</b>	$Y \sim P_o(13)$	B1	1	Poisson <b>and</b> 13																								
<b>(ii)</b>	$P(Y \geq 15) = 1 - P(Y < 14)$ $= 1 - 0.6751$ $= 0.3249$ $= 0.325$ $\therefore p = (0.3249)^4$ $p = 0.0111 \text{ to } 0.0112$	M1 A1✓ M1	4	On their $\lambda$ from b (i)  On their $p(Y \geq 15)$																								
<b>Total</b>			<b>7</b>																									
<b>2</b>	<p><math>H_0</math> : time of day has no effect on the outcome of a frame of snooker</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th><math>O_i</math></th> <th><math>E_i</math></th> <th><math> O_i - E_i  - 0.5</math></th> <th><math>\chi^2</math></th> </tr> </thead> <tbody> <tr> <td>30</td> <td>25.92</td> <td>3.58</td> <td>0.4945</td> </tr> <tr> <td>18</td> <td>22.08</td> <td>3.58</td> <td>0.5805</td> </tr> <tr> <td>24</td> <td>28.08</td> <td>3.58</td> <td>0.4564</td> </tr> <tr> <td>28</td> <td>23.92</td> <td>3.58</td> <td>0.5358</td> </tr> <tr> <td>100</td> <td>100</td> <td></td> <td>2.0672</td> </tr> </tbody> </table> <p><math>\chi^2_{5\%}(1) = 3.841</math>  <math>2.07 &lt; 3.841 \therefore</math> do not reject <math>H_0</math>                      No evidence to suggest that the time of day has an effect on the outcome of a frame of snooker played by Syd.</p>	$O_i$	$E_i$	$ O_i - E_i  - 0.5$	$\chi^2$	30	25.92	3.58	0.4945	18	22.08	3.58	0.5805	24	28.08	3.58	0.4564	28	23.92	3.58	0.5358	100	100		2.0672	B1 M1A1 M1 M1 A1 B1B1✓ A1✓ E1✓	10	<p><math>H_0</math> : outcome does not depend on time of day For E's</p> <p>For use of Yates' correction attempted calculation of <math>\chi^2</math> (even if Yates' correction not used)</p> <p>For <math>\nu = 1</math> and <math>\chi^2</math> On their <math>\chi^2</math></p>
$O_i$	$E_i$	$ O_i - E_i  - 0.5$	$\chi^2$																									
30	25.92	3.58	0.4945																									
18	22.08	3.58	0.5805																									
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28	23.92	3.58	0.5358																									
100	100		2.0672																									
<b>Total</b>			<b>10</b>																									
<b>3(a)</b>	$\sum x = 15.8$ $\sum x^2 = 25.0592$ $\bar{x} = \frac{15.8}{10} = 1.58$ $s^2 = \frac{25.0592}{9} - \frac{10}{9}(1.58)^2$ $= 0.01057$	B1 B2	3	$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{10}\right)$ <p>(AWRT 0.011) (<math>s = 0.1028</math>)</p>																								
<b>(b)</b>	<p>90% CI for <math>\mu</math></p> $1.58 \pm \frac{s}{\sqrt{10}} \times 1.833$ <p>(1.52, 1.64)</p>	M1A1 ft B1 B1✓ A1✓	5	$1.58 \pm 0.0596$  for $\nu = 9$ for t for interval																								
<b>Total</b>			<b>8</b>																									

**MS2B (cont)**

<b>Q</b>	<b>Solution</b>	<b>Marks</b>	<b>Total</b>	<b>Comments</b>
<b>4(a)</b>	$k = 0.1$	B1	1	OE.
<b>(b)</b>	$E(X) = 1$	B1	1	
<b>(c)</b>	$P(X > 0) = 6 \times 0.1$ $= 0.6$	M1 A1	2	
<b>(d)</b>	$P( X  > 3.5) = 1 - P( X  < 3.5)$ $= 1 - 0.7$ $= 0.3$	M1 A1 A1	3	
	Alternative solution $P(X < -3.5) + P(X > 3.5)$ $= \frac{0.5}{10} + \frac{2.5}{10}$ $= \frac{3}{10}$	(M1) (A1) (A1)		
	<b>Total</b>		<b>7</b>	

## MS2B (cont)

Q	Solution	Marks	Total	Comments								
5(a)	$E(R) = \left(1 \times \frac{1}{4}\right) + \left(2 \times \frac{1}{2}\right) + \left(4 \times \frac{1}{4}\right)$ $= 2.25$	M1A1	4	$2\frac{1}{4}$  $6\frac{1}{4}$  $1\frac{3}{16}$ (on their E (R))								
	$E(R^2) = \left(1 \times \frac{1}{4}\right) + \left(4 \times \frac{1}{2}\right) + \left(16 \times \frac{1}{4}\right)$ $= 6.25$ $\therefore \text{Var}(R) = 6.25 - (2.25)^2$ $= 1.1875$	M1 A1✓										
(b)(i)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td><math>x</math></td> <td>1</td> <td><math>\frac{1}{4}</math></td> <td><math>\frac{1}{16}</math></td> </tr> <tr> <td><math>P(X=x)</math></td> <td><math>\frac{1}{4}</math></td> <td><math>\frac{1}{2}</math></td> <td><math>\frac{1}{4}</math></td> </tr> </table>	$x$	1	$\frac{1}{4}$	$\frac{1}{16}$	$P(X=x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	B1	3	AG
	$x$	1	$\frac{1}{4}$	$\frac{1}{16}$								
$P(X=x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$									
$E(X) = \left(1 \times \frac{1}{4}\right) + \left(\frac{1}{4} \times \frac{1}{2}\right) + \left(\frac{1}{16} \times \frac{1}{4}\right)$ $= \frac{1}{4} + \frac{1}{8} + \frac{1}{64}$ $= \frac{16+8+1}{64}$ $= \frac{25}{64}$	M1  A1											
(ii)	$A = \left(R + \frac{8}{R}\right) \times \frac{8}{R} = 8 + \frac{64}{R^2}$	M1	3	(Attempt at area )								
	$E(A) = E\left(8 + \frac{64}{R^2}\right) = 8 + E\left(\frac{64}{R^2}\right)$ $= 8 + 64 \times E(X)$ $= 8 + 64 \times \frac{25}{64}$ $= 33$	M1  A1										
<b>Total</b>			<b>10</b>	CAO								

**MS2B (cont)**

<b>Q</b>	<b>Solution</b>	<b>Marks</b>	<b>Total</b>	<b>Comments</b>	
<b>6</b>	$H_0 : \mu = 568$	B1		$X \sim$ contents of cartons of milk $X \sim N(568, \sigma^2)$ Under $H_0$ : $\bar{X} \sim N\left(568, \frac{\sigma^2}{n}\right)$	
	$H_1 : \mu < 568$				
	1% one-tailed test	B1			
	$\nu = 7$				
	$\bar{x} = \frac{4510}{8} = 563.75$	B1			
	$\Rightarrow s^2 = \frac{254256.8}{7} - \frac{8}{7}(563.75)^2$				
	$s^2 = 7.929$	B2			$(s = 2.816)$
	$t = \frac{563.75 - 568}{2.816/\sqrt{8}}$	M1			
	$t = -4.27$	A1ft			$(AWFW = 4.27 \text{ to } -4.26)$
	$t_{crit} = -2.998$	B1ft			
reject $H_0$	A1✓	On their t			
Evidence at the 1% level of significance to suggest that the average contents of the cartons have been reduced.	E1✓				
	<b>Total</b>		<b>10</b>		

## MS2B (cont)

Q	Solution	Marks	Total	Comments
7(a)		B3	3	B1 2 axes with scales B1 horizontal line at 0.2 from 0 to 3 B1 curve from 3 to 6
(b)	$P(T = 3) = 0$	B1	1	
(c)	$P(T \geq 3) = 1 - P(T < 3)$ $= 1 - \frac{3}{5}$ $= \frac{2}{5}$	M1		$\int_3^6 \frac{1}{45} t(6-t) dt = \frac{2}{5}$
(d)	$\int_0^m \frac{1}{5} dt = 0.5$ $\left(\frac{t}{5}\right)_0^m = 0.5$ $\frac{m}{5} - 0 = 0.5$ $m = 0.5 \times 5$ $m = 2.5$	M1		$P(T \leq 3) = 0.6$ $\therefore 0 \leq \text{median} < 3$
(e)	$E(T) = \int_0^3 \frac{1}{5} t dt + \int_3^6 \frac{1}{45} t^2 (6-t) dt$ $= \left[\frac{1}{10} t^2\right]_0^3 + \left[\frac{2}{45} t^3 - \frac{1}{180} t^4\right]_3^6$ $= \frac{9}{10} + 1.65$ $= 2.55$ $\therefore P(\text{median} < T < \text{mean})$ $= P(2.5 < T < 2.55)$ $= 0.05 \times \frac{1}{5}$ $= 0.01$	M1		$\frac{1}{5} m = 0.5$ $m = 5 \times 0.5$ $m = 2.5$ AG
	<b>Total</b>		<b>6</b>	
			<b>14</b>	





## MS2B (cont)

Q	Solution	Marks	Total	Comments
8(a)	$H_0 : \mu = 35$ $H_1 : \mu \neq 35$  2 – tail test , 1% sig. level  under $H_0$ , $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$  $\bar{X} \sim N\left(35, \frac{144}{100}\right)$  $z = \frac{37.9 - 35}{1.2}$  $z = 2.42$  $z_{crit} = \pm 2.5758$  do not reject $H_0$  Evidence to support the claim that the mean age is 35 years.	B1     B1  M1  A1✓  B1  A1✓  E1✓	7	$z = \frac{37.9 - 35}{\text{their } \sigma/\sqrt{n}}$  On their $\sigma/\sqrt{n}$    On their z
(b)	Accept $H_0$ when $H_0$ false Accepting the mean to be 35years when it isn't.	B2	2	Allow B1 if not in context
	<b>Total</b>		<b>9</b>	
	<b>Total</b>		<b>75</b>	