

General Certificate of Education
June 2006
Advanced Level Examination



MATHEMATICS
Unit Statistics 2A

MS2A/W

Wednesday 24 May 2006 1.30 pm to 2.45 pm

For this paper you must have:

- an 8-page answer book
 - the **blue** AQA booklet of formulae and statistical tables
- You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MS2A/W.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 60.
- The marks for questions are shown in brackets.
- Unit Statistics 2A has a **written paper and coursework**.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1** The number of A-grades, X , achieved in total by students at Lowkey School in their Mathematics examinations each year can be modelled by a Poisson distribution with a mean of 3.
- (a) Determine the probability that, during a 5-year period, students at Lowkey School achieve a total of more than 18 A-grades in their Mathematics examinations. *(3 marks)*
- (b) The number of A-grades, Y , achieved in total by students at Lowkey School in their English examinations each year can be modelled by a Poisson distribution with a mean of 7.
- (i) Determine the probability that, during a year, students at Lowkey School achieve a total of fewer than 15 A-grades in their Mathematics and English examinations. *(3 marks)*
- (ii) What assumption did you make in answering part (b)(i)? *(1 mark)*
- 2** It is claimed that the area within which a school is situated affects the age profile of the staff employed at that school. In order to investigate this claim, the age profiles of staff employed at two schools with similar academic achievements are compared.

Academia High School, situated in a rural community, employs 120 staff whilst Best Manor Grammar School, situated in an inner-city community, employs 80 staff.

The **percentage** of staff within each age group, for each school, is given in the table.

Age	Academia High School	Best Manor Grammar School
22–34	17.5	40.0
35–39	60.0	45.0
40–59	22.5	15.0

- (a) (i) Form the data into a contingency table suitable for analysis using a χ^2 distribution. *(2 marks)*
- (ii) Use a χ^2 test, at the 1% level of significance, to determine whether there is an association between the age profile of the staff employed and the area within which the school is situated. *(9 marks)*
- (b) Interpret your result in part (a)(ii) as it relates to the 22–34 age group. *(2 marks)*

- 3 The number of aces, X , served by a tennis player during a set can be modelled by the following probability distribution.

x	0	1	2	3	4	5
$P(X=x)$	0.01	0.09	0.13	0.50	0.15	0.12

- (a) Calculate the mean, μ , and the standard deviation, σ , of X . *(4 marks)*
- (b) For a set selected at random, calculate $P(\mu - \sigma < X < \mu + \sigma)$. *(3 marks)*
- 4 The error, R centilitres, made in measuring the contents of bottles of wine can be modelled by the rectangular distribution

$$f(r) = \begin{cases} \alpha & -0.4 < r < 0.4 \\ 0 & \text{otherwise} \end{cases}$$

where α is a positive constant.

- (a) Find the value of α . *(1 mark)*
- (b) Calculate the mean and the standard deviation of R . *(3 marks)*
- (c) For a bottle of wine selected at random, calculate the probability that the **magnitude** of the error made in measuring its contents is less than 0.3 cl. *(2 marks)*

Turn over for the next question

5 The lifetime, X hours, of Everwhite camera batteries is normally distributed. The manufacturer claims that the mean lifetime of these batteries is 100 hours.

- (a) The members of a photography club suspect that the batteries do not last as long as is claimed by the manufacturer. In order to investigate their suspicion, the members test a random sample of five of these batteries and find the lifetimes, in hours, to be as follows:

85 92 100 95 99

Test the members' suspicion at the 5% level of significance. (9 marks)

- (b) The manufacturer, believing that the mean lifetime of these batteries has not changed from 100 hours, decides to determine the lifetime, x hours, of each of a random sample of 80 Everwhite camera batteries. The manufacturer obtains the following results, where \bar{x} denotes the sample mean:

$$\sum x = 8080 \quad \text{and} \quad \sum (x - \bar{x})^2 = 6399$$

Test the manufacturer's belief at the 5% level of significance. (8 marks)

6 The continuous random variable X has probability density function defined by

$$f(x) = \begin{cases} \frac{1}{5}(2x + 1) & 0 \leq x \leq 1 \\ \frac{1}{15}(4 - x)^2 & 1 < x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Sketch the graph of f . (2 marks)

- (b) (i) Show that the cumulative distribution function, $F(x)$, for $0 \leq x \leq 1$ is

$$F(x) = \frac{1}{5}x(x + 1) \quad (3 \text{ marks})$$

- (ii) Hence find $P(X \geq 0.5)$. (2 marks)

- (iii) Verify that the lower quartile, q_1 , satisfies

$$0.5 < q_1 < 0.75 \quad (3 \text{ marks})$$

END OF QUESTIONS