

General Certificate of Education
June 2006
Advanced Level Examination



MATHEMATICS
Unit Statistics 3

MS03

Wednesday 21 June 2006 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MS03.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 A council claims that 80 per cent of households are generally satisfied with the services it provides.

A random sample of 250 households shows that 209 are generally satisfied with the council's provision of services.

- (a) Construct an approximate 95% confidence interval for the proportion of households that are generally satisfied with the council's provision of services. *(6 marks)*
- (b) Hence comment on the council's claim. *(2 marks)*

- 2 The table below shows the heart rates, x beats per minute, and the systolic blood pressures, y milligrams of mercury, of a random sample of 10 patients undergoing kidney dialysis.

Patient	1	2	3	4	5	6	7	8	9	10
x	83	86	88	92	94	98	101	111	115	121
y	157	172	161	154	171	169	179	180	192	182

- (a) Calculate the value of the product moment correlation coefficient for these data. *(3 marks)*
- (b) Assuming that these data come from a bivariate normal distribution, investigate, at the 1% level of significance, the claim that, for patients undergoing kidney dialysis, there is a positive correlation between heart rate and systolic blood pressure. *(4 marks)*

- 3 Each enquiry received by a business support unit is dealt with by Ewan, Fay or Gaby. The probabilities of them dealing with an enquiry are 0.2, 0.3 and 0.5 respectively.

Of enquiries dealt with by Ewan, 60% are answered immediately, 25% are answered later the same day and the remainder are answered at a later date.

Of enquiries dealt with by Fay, 75% are answered immediately, 15% are answered later the same day and the remainder are answered at a later date.

Of enquiries dealt with by Gaby, 90% are answered immediately and the remainder are answered at a later date.

- (a) Determine the probability that an enquiry:
- (i) is dealt with by Gaby and answered immediately; (1 mark)
 - (ii) is answered immediately; (3 marks)
 - (iii) is dealt with by Gaby, given that it is answered immediately. (3 marks)
- (b) Determine the probability that an enquiry is dealt with by Ewan, given that it is answered later the same day. (4 marks)

- 4 The table below shows the probability distribution for the number of students, R , attending classes for a particular mathematics module.

r	6	7	8
$P(R=r)$	0.1	0.6	0.3

- (a) Find values for $E(R)$ and $\text{Var}(R)$. (4 marks)
- (b) The number of students, S , attending classes for a different mathematics module is such that

$$E(S) = 10.9, \quad \text{Var}(S) = 1.69 \quad \text{and} \quad \rho_{RS} = \frac{2}{3}$$

Find values for the mean and variance of:

- (i) $T = R + S$; (4 marks)
- (ii) $D = S - R$. (2 marks)

Turn over for the next question

- 5 The number of letters per week received at home by Rosa may be modelled by a Poisson distribution with parameter 12.25.
- (a) Using a normal approximation, estimate the probability that, during a 4-week period, Rosa receives at home at least 42 letters but at most 54 letters. *(5 marks)*
- (b) Rosa also receives letters at work. During a 16-week period, she receives at work a total of 248 letters.
- (i) Assuming that the number of letters received at work by Rosa may also be modelled by a Poisson distribution, calculate a 98% confidence interval for the average number of letters per week received at work by Rosa. *(5 marks)*
- (ii) Hence comment on Rosa's belief that she receives, on average, fewer letters at home than at work. *(2 marks)*

- 6 The random variable X has a Poisson distribution with parameter λ .
- (a) Prove that $E(X) = \lambda$. *(3 marks)*
- (b) By first proving that $E(X(X - 1)) = \lambda^2$, or otherwise, prove that $\text{Var}(X) = \lambda$. *(5 marks)*

- 7 A shop sells cooked chickens in two sizes: medium and large.

The weights, X grams, of medium chickens may be assumed to be normally distributed with mean μ_X and standard deviation 45.

The weights, Y grams, of large chickens may be assumed to be normally distributed with mean μ_Y and standard deviation 65.

A random sample of 20 medium chickens had a mean weight, \bar{x} grams, of 936.

A random sample of 10 large chickens had the following weights in grams:

1165 1202 1077 1144 1195 1275 1136 1215 1233 1288

- (a) Calculate the mean weight, \bar{y} grams, of this sample of large chickens. *(1 mark)*
- (b) Hence investigate, at the 1% level of significance, the claim that the mean weight of large chickens exceeds that of medium chickens by more than 200 grams. *(8 marks)*
- (c) (i) Deduce that, for your test in part (b), the critical value of $(\bar{y} - \bar{x})$ is 253.24, correct to two decimal places. *(2 marks)*
- (ii) Hence determine the power of your test in part (b), given that $\mu_Y - \mu_X = 275$. *(5 marks)*
- (iii) Interpret, in the context of this question, the value that you obtained in part (c)(ii). *(3 marks)*

END OF QUESTIONS