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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MS03 Statistics 3

Mark Scheme

2007 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Dr Michael Cresswell Director General

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS03

Q	Solution	Marks	Total	Comments
1(a)	Samples are independent or random	B1		
	98% $\Rightarrow$ z = 2.3263	B1		AWFW 2.32 to 2.33
	CI for $\mu_1 - \mu_2$ is:			
	$(\bar{x}_S - \bar{x}_A) \pm z \times \sqrt{\frac{s_S^2}{n_S} + \frac{s_A}{n_A}}$	M1 A1		Form Allow: sigmas, A&B or 1&2 and $n-1$ Correct
	$(19268 - 17896)$ $\pm 2.3263 \times \sqrt{\frac{7321^2}{175} + \frac{8205^2}{225}}$	A1√		√ on z only s_P = 7830 to 7850
	ie 1372 ± (1805 to 1820) or (–450 to –430, 3170 to 3200)	A1	6	1372 ± (1830 to 1845) AWFW
(b)	Confidence interval includes zero so (at 5% level) Mean starting salaries may be equal	B1√ ↑dep↑ B1√	 2	√ on CI; OE √ on CI; OE
	Total		**8**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$P(\geq 18 \mid \text{Road}) = 0.85$	B1	1	CAO; OE; **not** 85
(b)	$P(18 \text{ to } 64) =$ $P(\text{Route}) \times P(18 \text{ to } 64 \mid \text{Route}) =$	M1		Use of 3 possibilities, each the product of 2 probabilities
	$(0.25 \times 0.80)+(0.60 \times 0.35)+(0.55 \times 0.40)$	A1		At least 1 term correct
	$= 0.20 + 0.21 + 0.22 = 0.63$	A1	3	CAO; OE
(c)	$P(\text{FR} \cap >64) = P(\text{FR}) \times P(>64 \mid \text{FR})$			
	$= 0.35 \times 0.15$	B1		Correct expression
	$= 0.052$ to 0.053	B1	2	AWFW (0.0525)
(d)	$P(\text{FR} \mid >64) = \frac{(c)}{P(>64)} =$	M1 M1		$\frac{\text{answer}(c)}{\sum(3\times 2) \text{ probabilities}}$
	$\frac{0.0525}{(0.25\times 0.05)+(0.35\times 0.15)+(0.40\times 0.35)}$	A1		At least 2 terms correct
	$= \frac{0.0525}{0.0125+0.0525+0.1400} = \frac{0.0525}{0.205}$	A1		CAO
	$= 0.256$ or $\frac{21}{82}$	A1	5	AWRT/CAO; OE
	Total		**11**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$H_0: p_K = p_S$ $H_1: p_K \neq p_S$	B1		Both; OE; allow A&B or 1&2
	SL $\alpha = 0.05$ CV $\lvert z \rvert = 1.96$	B1		CAO
	$\hat{p} = \dfrac{(150\times0.28)+(250\times0.34)}{400}$	M1		Used
	$= \dfrac{127}{400}$ or 0.317 to 0.318	A1		CAO/AWFW (0.3175)
	$z = \dfrac{(\hat{p}_K - \hat{p}_S)-0}{\sqrt{\hat{p}(1-\hat{p})\left(\dfrac{1}{n_K}+\dfrac{1}{n_S}\right)}}$	M1		Used; accept unpooled denominator
	$\lvert z \rvert = \dfrac{\lvert 0.28-0.34 \rvert}{\sqrt{0.3175\times0.6825\left(\dfrac{1}{150}+\dfrac{1}{250}\right)}}$	A1√		√ on $\hat{p}$; accept no pooling
	$=$ |1.24| to |1.25|	A1		AWFW; |1.26| to |1.27|
	Thus accept H_0 as $\lvert z \rvert < 1.96$	A1√		√ on z and CV with same sign
	Thus no evidence, at 5% level, of a difference between two proportions of male customers in two salons	E1√	9	√ on z and CV with same sign In context and qualified
(b)	Zero since	B1		CAO
	Cannot make a Type I error when H_0 is false	B1	2	OE
	Total		**11**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
4	98% $\Rightarrow$ $z = 2.5758$	B1		AWFW 2.57 to 2.58
	CI width is $2\times\frac{z\sigma}{\sqrt{n}}$	M1		Used; allow $\frac{z\sigma}{\sqrt{n}}$
	Thus $2\times\frac{2.5758\times0.08}{\sqrt{n}} = 0.05$	A1√		OE; √ on z; allow no '2 ×'
	Thus $\sqrt{n} = 8.24256$	m1		Solving for $\sqrt{n}$ or n
	Thus $n = 67.9 \Rightarrow 68$	A1√		AWRT; √ on z
	Thus, to nearest 5, $n = 70$	A1	6	CAO
	Total		**6**	
5	$D = \sum X_i - \sum Y_i$ or $D' = \sum Y_i - \sum X_i$	M1		Used or implied
	have means $\mu = 162 - 166 = -4$ $\mu = 166 - 162 = +4$	B1		CAO either
	and variance $\sigma^2 = (3\times2^2) + (2\times3^2) = 12 + 18$ $= 30$	M1 A1		Use of [a × Var(Z)]; implied CAO
	$P\left(\sum X_i < \sum Y_i\right) =$			
	$P(D<0)$ or $P(D'>0) =$	M1		Used or implied
	$P\left(Z > \frac{0-(-4)}{\sqrt{30}}\right)$ or $P\left(Z > \frac{0-(+4)}{\sqrt{30}}\right) =$	m1		Standardising 0 using μ and $\sqrt{\sigma^2}$
	$P(Z < +0.73)$ or $P(Z > -0.73) =$			
	0.767 to 0.768	A1	7	AWFW
	Total		**7**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\mathrm{E}(X) = \sum_{x=0}^{n} x \times \binom{n}{x} p^x (1-p)^{n-x}$	M1		Use of $\sum x \times \mathrm{P}(X = x)$
	$= \sum_{x=1}^{n} \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x}$	M1		Expansion of ${}^nC_x$; cancelling of x (Ignore limits)
	$= np \times \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} (1-p)^{n-x}$	M1		Factors of n and p (Ignore limits)
	$= np \times \sum \mathrm{P}(X = x) \mid \mathrm{B}(n-1,\ p) = np$	M1	4	AG; must be convincing
(ii)	$\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$	M1		Used
	$= [\mathrm{E}(X^2) - \mathrm{E}(X)] + \mathrm{E}(X) - (\mathrm{E}(X))^2$ $= n(n-1)p^2 + np - n^2p^2$	m1		Attempted
	$= np(1-p)$	A1	3	AG; must be convincing
(iii)	Thus $np(1-p) = 3(1-p) = 2.97$	M1		Substituting μ in σ^2
	Thus $1-p = \frac{2.97}{3} = 0.99$			
	Thus $p = 0.01$	A1		CAO
	and $n = 300$	A1	3	CAO
(iv)	$\mathrm{B}(300,\ 0.01) \sim \mathrm{Po}(3)$	B1		CAO; PI
	$\mathrm{P}(X > 2) = 1 - \mathrm{P}(X \leq 2)$	M1		Must be applied to Poisson
	$= 1 - 0.4232 = 0.577$	A1	3	AWRT

MS03 (cont)

Q	Solution	Marks	Total	Comments
6(a)			13	
(b)	$Y \sim$ B(500, 0.45) or $Y \sim$ (normal) with mean $\mu = 225$	B1		PI
	and variance $\sigma^2 = 123.75$			AWFW 123 to 124
	or standard deviation $\sigma = 11.124$	B1		AWFW 11.05 to 11.15
	(At least) half $\Rightarrow$ ($\geq$) 250	B1		CAO
	$P(Y_B \geq 250) = P(Y_N > 249.5) =$	B1		CAO
	$P\left(Z > \frac{249.5 - 225}{\sqrt{123.75}}\right) =$	M1		Standardising 249.5, 250 or 250.5 with c's μ and $\sqrt{\sigma^2}$
	$P(Z > 2.20) = 1 - P(Z < 2.20)$	m1		Area change
	= 0.0138 to 0.014	A1	7	
	Note:			
	Use of $\frac{0.5 - 0.45}{\sqrt{0.000495}} \Rightarrow$ max of 5 marks			Use of distribution of $\hat{p}$
	Use of $\frac{0.499 - 0.45}{\sqrt{0.000495}} \Rightarrow$ max of 7 marks			Use of distribution of $\hat{p}$ with continuity correction
	Total		**20**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$H_0 : \lambda = 13$	B1		CAO; OE
	$H_1 : \lambda < 13$	B1		CAO; OE
	$P(R \leq 10 \mid Po(13)$	M1		Used or implied
	$= 0.2517$	A1		AWFW 0.251 to 0.252
	Prob of $0.2517 > 0.10$ (10%) $z = -0.83$ to $-0.70 > -1.28$	M1		Comparison of prob with 0.10 Comparison of z with -1.28
	Thus no evidence, at 10% level, of a reduction in the mean value of R	A1√	6	√ on probability or z In 'context' and qualified
(b)	Require $P(R \leq r \mid Po(13)) \approx 0.10$	M1		Stated or implied
	Critical Region is $R \leq 8$ or $R < 9$	A1	2	Accept $R = 8$ May be scored in (a)
(c)	Require P(accept $H_0 \mid H_0$ false)	B1		OE; PI
	$= P(R > 8 \mid Po(6.5))$	M1		Use of Po(6.5)
	$= 1 - P(R \leq 8 \mid Po(6.5))$	m1		
	$= 1 - 0.7916$			
	$= 0.208$ to 0.209	A1	4	AWFW (0.2084)
	Total		**12**	
	TOTAL		**75**	