

General Certificate of Education
June 2008
Advanced Level Examination



MATHEMATICS
Unit Pure Core 4

MPC4

Thursday 12 June 2008 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The polynomial $f(x)$ is defined by $f(x) = 27x^3 - 9x + 2$.

(a) Find the remainder when $f(x)$ is divided by $3x + 1$. (2 marks)

(b) (i) Show that $f\left(-\frac{2}{3}\right) = 0$. (1 mark)

(ii) Express $f(x)$ as a product of three linear factors. (4 marks)

(iii) Simplify

$$\frac{27x^3 - 9x + 2}{9x^2 + 3x - 2} \quad (2 \text{ marks})$$

2 A curve is defined, for $t \neq 0$, by the parametric equations

$$x = 4t + 3, \quad y = \frac{1}{2t} - 1$$

At the point P on the curve, $t = \frac{1}{2}$.

(a) Find the gradient of the curve at the point P . (4 marks)

(b) Find an equation of the normal to the curve at the point P . (3 marks)

(c) Find a cartesian equation of the curve. (3 marks)

3 (a) By writing $\sin 3x$ as $\sin(x + 2x)$, show that $\sin 3x = 3 \sin x - 4 \sin^3 x$ for all values of x . (5 marks)

(b) Hence, or otherwise, find $\int \sin^3 x \, dx$. (3 marks)

4 (a) (i) Obtain the binomial expansion of $(1 - x)^{\frac{1}{4}}$ up to and including the term in x^2 . (2 marks)

(ii) Hence show that $(81 - 16x)^{\frac{1}{4}} \approx 3 - \frac{4}{27}x - \frac{8}{729}x^2$ for small values of x . (3 marks)

(b) Use the result from part (a)(ii) to find an approximation for $\sqrt[4]{80}$, giving your answer to seven decimal places. (2 marks)

- 5 (a) The angle α is acute and $\sin \alpha = \frac{4}{5}$.
- (i) Find the value of $\cos \alpha$. (1 mark)
- (ii) Express $\cos(\alpha - \beta)$ in terms of $\sin \beta$ and $\cos \beta$. (2 marks)
- (iii) Given also that the angle β is acute and $\cos \beta = \frac{5}{13}$, find the exact value of $\cos(\alpha - \beta)$. (2 marks)
- (b) (i) Given that $\tan 2x = 1$, show that $\tan^2 x + 2 \tan x - 1 = 0$. (2 marks)
- (ii) Hence, given that $\tan 45^\circ = 1$, show that $\tan 22\frac{1}{2}^\circ = \sqrt{2} - 1$. (3 marks)
- 6 (a) Express $\frac{2}{x^2 - 1}$ in the form $\frac{A}{x - 1} + \frac{B}{x + 1}$. (3 marks)
- (b) Hence find $\int \frac{2}{x^2 - 1} dx$. (2 marks)
- (c) Solve the differential equation $\frac{dy}{dx} = \frac{2y}{3(x^2 - 1)}$, given that $y = 1$ when $x = 3$.
- Show that the solution can be written as $y^3 = \frac{2(x - 1)}{x + 1}$. (5 marks)
- 7 The coordinates of the points A and B are $(3, -2, 1)$ and $(5, 3, 0)$ respectively.
- The line l has equation $\mathbf{r} = \begin{bmatrix} 5 \\ 3 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$.
- (a) Find the distance between A and B . (2 marks)
- (b) Find the acute angle between the lines AB and l . Give your answer to the nearest degree. (5 marks)
- (c) The points B and C lie on l such that the distance AC is equal to the distance AB . Find the coordinates of C . (5 marks)

Turn over for the next question

- 8 (a) The number of fish in a lake is decreasing. After t years, there are x fish in the lake. The rate of decrease of the number of fish is proportional to the number of fish currently in the lake.
- (i) Formulate a differential equation, in the variables x and t and a constant of proportionality k , where $k > 0$, to model the rate at which the number of fish in the lake is decreasing. (2 marks)
- (ii) At a certain time, there were 20 000 fish in the lake and the rate of decrease was 500 fish per year. Find the value of k . (2 marks)
- (b) The equation

$$P = 2000 - Ae^{-0.05t}$$

is proposed as a model for the number of fish, P , in another lake, where t is the time in years and A is a positive constant.

On 1 January 2008, a biologist estimated that there were 700 fish in this lake.

- (i) Taking 1 January 2008 as $t = 0$, find the value of A . (1 mark)
- (ii) Hence find the year during which, according to this model, the number of fish in this lake will first exceed 1900. (4 marks)

END OF QUESTIONS