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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MPC4 Pure Core 4

Mark Scheme

2005 examination – June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	OE	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

Application of Mark Scheme

No method shown:	
Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

MPC4

Q	Solution	Marks	Total	Comments
1(a)	$R=\sqrt{5}$ $\left(\text{or } \sqrt{1+2^2} \text{ or } 2.23 \text{ or } 2.24\right)$	B1		
	$\frac{\sin\alpha}{\cos\alpha}=\frac{1}{2}$ $\alpha=26.6^\circ$	M1A1	3	CAO **SC** 63.4º 1/2
(b)	$\sin(x+26.6)=\frac{1}{\sqrt{5}}$	M1		
	$x=\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)-26.6$	m1		
	$x=0°$ $x=126.8°$(126.9º, or 127 or 126º with working)	B1 A1	4	B1 $x=0^\circ$ lose if extra solution in range **SC** calculator trace:126.9 full marks **SC** ft from 63.4º
	Total		**7**	
2 (a)	$3x-5=A(2x-1)+B(x+3)$	M1		
	$x=-3$ $A=2$, $x=\frac{1}{2}$ $B=-1$	m1A1	3	ml: sub in 2 values or set up simultaneous equations
(b)	$\int\left(\frac{2}{x+3}-\frac{1}{2x-1}\right)\mathrm{d}x$	M1		
	$=2\ln(x+3)-\frac{1}{2}\ln(2x-1)(+c)$	m1 A1√	3	Use of lns for either integral ft A and B
	Total		**6**	
3 (a)	$\mathrm{f}\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3-\left(\frac{1}{2}\right)^2+2\left(\frac{1}{2}\right)-2=-1$	M1A1	2	Allow M1 for $\mathrm{f}\left(-\frac{1}{2}\right)$
(b)	$\frac{x^2(2x-1)}{2x-1}+\frac{2x-2}{2x-1}$	M1M1		Long division: M1for their complete attempt Reasonable start, complete method Could be done by long division, which may have been done in (a)
	$x^2+\frac{2x-1-1}{2x-1}=x^2+1-\frac{1}{2x-1}$	AlBl√	4	$a=$ 1 CAO $b=-1$ ft part (a)
	Total		**6**	
4 (a)	$(1+x)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)x+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}x^2$	M1A1	2	$1-\frac{1}{2}x+\frac{3}{8}x^2$ but simplification not require
(b)	$\frac{1}{\sqrt{1+2x}}=(1+2x)^{-\frac{1}{2}}$	B1		
	$=1-\frac{1}{2}(2x)+\frac{3}{8}(2x)^2$	M1		Condone missing brackets, if recovered
	$=1-x+\frac{3}{2}x^2$	A1	3	CAO
(c)	$1-(-0.1)+\frac{3}{2}(-0.1)^2$ $(=1.115)$	M1		Attempt to substitute in
	$(1-0.2)^{-\frac{1}{2}}=\frac{\sqrt{5}}{2}$;	M1		Link between $\frac{1}{\sqrt{1+2x}}$ and $\frac{\sqrt{5}}{2}$
	$2\times1.115=2.23\approx\sqrt{5}$	A1	3	AG; convincingly obtained
	Total		**8**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
5 (a)	$t=\frac{1}{2}\quad x=3\quad y=2$	B1B1	2	
(b)	$t=\frac{1}{y}\qquad x=2\frac{1}{y}+y$	M1		Attempt to eliminate t **SC** verification using t 1/2
	$xy=2+y^2\qquad xy-y^2=2$	A1	2	AG; convincingly found
(c)	Implicit differentiation:	M1		Attempt at equation with
	$x\frac{dy}{dx}+y,-2y\frac{dy}{dx}=0$	A1A1		$\frac{dy}{dx}$ but not "$\frac{dy}{du}$=..."
		B1		RHS = 0
	At (3, 2) $\quad 3\frac{dy}{dx}+2-4\frac{dy}{dx}=0$	m1		Use of (3, 2)
	$\frac{dy}{dx}=2$	A1	6	AG; convincingly obtained
	OR Parametric differentiation:			
	$\frac{dy}{dx}=\frac{dy}{dt}\frac{1}{\frac{dx}{dt}}=-\frac{1}{t^2}\frac{1}{2-\frac{1}{t^2}}$	(M1) (A1A1)		Attempt chain rule, PI
	$\frac{1}{t^2}=y^2$	(B1)		Or sub t $=\frac{1}{2}$; –4 in numerator is sufficient for this
	$y=2\qquad\frac{dy}{dx}=\frac{-4}{2-4}=2$	M1A1	6	AG; convincingly obtained
	OR $x=y+2y^{-1}$ M1 $\frac{dx}{dy}=1-2y^{-2}$ M1A1A1 $=1-\frac{1}{2}=\frac{1}{2}$ B1 $\rightarrow\frac{dy}{dx}=2$ A1			
	Total		**10**	
6 (a)	$\sin 2x=2\sin x\cos x$	B1	1	
(b) (i)	$A=B=x$	M1		
	$\cos 2x=\cos^2 x-\sin^2 x$	A1	2	
(ii)	$\cos(2x+x)=\cos 2x\cos x-\sin 2x\sin x$	M1		
	$=(\cos^2 x-\sin^2 x)\cos x-2\sin^2 x\cos x$	A1√		ft (a)and (b)(i)
	$=\cos^3 x-3(1-\cos^2 x)\cos x$	M1		Eliminate all sines
	$=4\cos^3 x-3\cos x$	A1	4	AG; convincingly obtained
(c)	$\cos^3 x=\frac{1}{4}(\cos 3x+3\cos x)$	M1		Ignore middle sign
	$\int_0^{\frac{\pi}{2}}\cos^3 x dx=\frac{1}{4}\left[\frac{1}{3}\sin 3x+3\sin x\right]_0^{\frac{\pi}{2}}$	A1A1		1 for integrity each term
		M1		Use of limits
	$\frac{1}{4}\left(-\frac{1}{3}+3\right)=\frac{2}{3}$	A1	5	AG
	Total		**12**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
7 (a)	$\lvert\overrightarrow{AB}\rvert = \sqrt{(2-1)^2 + (-1-4)^2 + (3-2)^2}$	M1		OE
	$= \sqrt{27} = 3\sqrt{3}$	A1	2	AG; convincingly obtained
(b)	$\begin{bmatrix} 1 \\ -5 \\ 1 \end{bmatrix} \bullet \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = 1+5+1$	M1		Their $\overrightarrow{AB} \bullet \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$
		M1		Their scalar product $= \begin{vmatrix} 1 \\ -1 \\ 1 \end{vmatrix} 3\sqrt{3}\cos\theta$
	$7 = 3\sqrt{3}\sqrt{3}\cos\theta \qquad \cos\theta = \frac{7}{9}$	A1	3	AG; convincingly obtained
(c)(i)	$\overrightarrow{OP} = \begin{bmatrix} 2+p \\ -1-p \\ 3+p \end{bmatrix} \qquad \overrightarrow{AP} = \begin{bmatrix} 1+p \\ -5-p \\ 1+p \end{bmatrix}$	M1A1		Finding $\overrightarrow{AP}$
	$\overrightarrow{AP} \bullet \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = (1+p)-(-5-p)+(1+p)$	m1		
	$= 7 + 3p$	A1	4	AG **SC** working with λ instead of p giving $7 + 3\lambda$ 3/4
(ii)	$7 + 3p = 0$	M1		
	$p = -\frac{7}{3} \qquad P$ is $\left(-\frac{1}{3}, \frac{4}{3}, \frac{2}{3}\right)$	A1A1	3	Allow column vectors and decimals
	Total		**12**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$t = 0 \quad x = 85$	B1	1	
(ii)	$t = 30 \quad x = 15 + 70e^{-\frac{30}{40}}$	M1		
	$= 48.06 \approx 48°C$	A1	2	
(iii)	$x = 60 \quad \frac{60-15}{70} = e^{-\frac{t}{40}}$	M1		
	$\ln\left(\frac{45}{70}\right) = -\frac{t}{40}$	m1		Their 60–15
	$t = 17.67 \approx 18$ minutes	A1	3	
(b)(i)	$\int \frac{dx}{x-15} = -\int \frac{dt}{40}$	M1 A1		Attempt to separate and integrate Correct expression and used
	$\ln(x-15) = -\frac{t}{40}(+c)$	A1A1		
	(0, 85) $c = \ln 70$	m1		Use (0.85) to find c, which must now appear in expression
	$\frac{t}{40} = \ln 70 - \ln(x-15) \quad t = 40\ln\left(\frac{70}{x-15}\right)$	A1	6	Manipulate to $t = \ldots$.
(ii)	$e^{\frac{t}{40}} = \frac{70}{x-15}$	M1		Manipulate expression including a 1n towards $x = \ldots$
	$x - 15 = 70e^{-\frac{t}{40}}$	M1A1	2	AG; convincingly obtained
	Total		**14**	
	Total		**75**	