

General Certificate of Education  
June 2005  
Advanced Level Examination



**MATHEMATICS**  
**Unit Pure Core 3**

**MPC3**

Friday 24 June 2005 Morning Session

**In addition to this paper you will require:**

- an 8-page answer book;
  - the **blue** AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

**Information**

- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

**Advice**

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

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Answer **all** questions.

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- 1 (a) Find  $\frac{dy}{dx}$  when  $y = x \sin 2x$ . *(3 marks)*
- (b) (i) Find  $\frac{dy}{dx}$  when  $y = (x^2 - 6)^4$ . *(2 marks)*
- (ii) Hence, or otherwise, find  $\int x(x^2 - 6)^3 dx$ . *(3 marks)*

- 2 The functions  $f$  and  $g$  are defined with their respective domains by

$$f(x) = x - 2 \quad \text{for all real values of } x$$

$$g(x) = \frac{6}{x + 3} \quad \text{for real values of } x, \quad x \neq -3$$

The composite function  $fg$  is denoted by  $h$ .

- (a) Find  $h(x)$ . *(2 marks)*
- (b) (i) Find  $h^{-1}(x)$ , where  $h^{-1}$  is the inverse of  $h$ . *(3 marks)*
- (ii) Find the range of  $h^{-1}$ . *(1 mark)*
- 3 (a) Find  $\int e^{4x} dx$ . *(1 mark)*
- (b) Use integration by parts to find  $\int e^{4x}(2x + 1) dx$ . *(3 marks)*
- (c) By using the substitution  $u = 1 + \ln x$ , or otherwise, find  $\int \frac{1 + \ln x}{x} dx$ . *(4 marks)*

4 It is given that  $\tan^2 x = \sec x + 11$ .

(a) Show that the equation  $\tan^2 x = \sec x + 11$  can be written in the form

$$\sec^2 x - \sec x - 12 = 0 \quad (2 \text{ marks})$$

(b) Hence show that  $\cos x = \frac{1}{4}$  or  $\cos x = -\frac{1}{3}$ . (3 marks)

(c) Hence, or otherwise, solve the equation  $\tan^2 x = \sec x + 11$ , giving all values of  $x$  to the nearest degree in the interval  $0^\circ < x < 360^\circ$ . (3 marks)

5 (a) Solve the equation  $2e^x = 5$ , giving your answer as an exact natural logarithm.

(2 marks)

(b) (i) By substituting  $y = e^x$ , show that the equation  $2e^x + 5e^{-x} = 7$  can be written as

$$2y^2 - 7y + 5 = 0 \quad (2 \text{ marks})$$

(ii) Hence solve the equation  $2e^x + 5e^{-x} = 7$ , giving your answers as exact values of  $x$ . (3 marks)

6 (a) (i) Sketch the graph of  $y = 4 - x^2$ , indicating the coordinates of the points where the graph crosses the coordinate axes. (2 marks)

(ii) The region between the graph and the  $x$ -axis from  $x = 0$  to  $x = 2$  is rotated through  $360^\circ$  about the  $x$ -axis. Find the exact value of the volume of the solid generated. (4 marks)

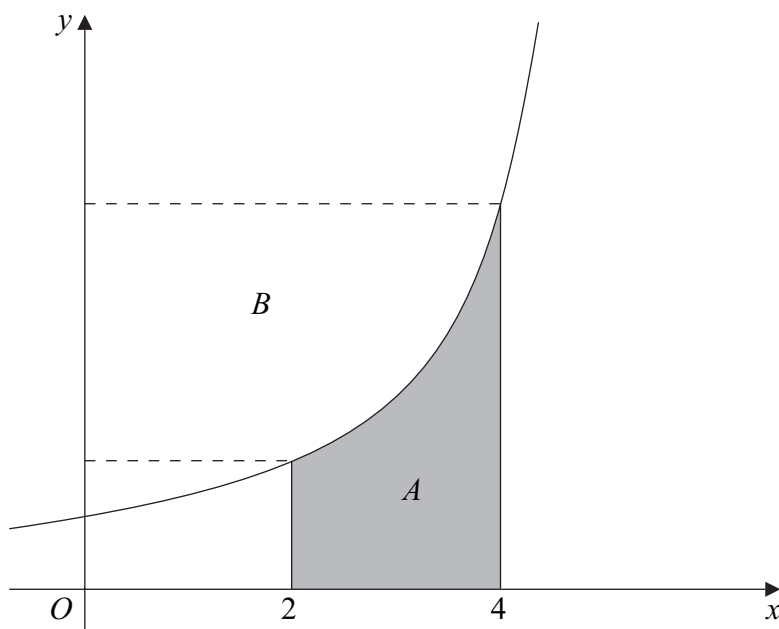
(b) (i) Sketch the graph of  $y = |4 - x^2|$ . (2 marks)

(ii) Solve  $|4 - x^2| = 3$ . (3 marks)

(iii) Hence, or otherwise, solve the inequality  $|4 - x^2| < 3$ . (2 marks)

- 7 (a) Sketch the graph of  $y = \tan^{-1}x$ . (2 marks)
- (b) (i) By drawing a suitable straight line on your sketch, show that the equation  $\tan^{-1}x = 2x - 1$  has only one root. (2 marks)
- (ii) Given that the root of this equation is  $\alpha$ , show that  $0.8 < \alpha < 0.9$ . (2 marks)
- (c) Use the iteration  $x_{n+1} = \frac{1}{2}(\tan^{-1}x_n + 1)$  with  $x_1 = 0.8$  to find the value of  $x_3$ , giving your answer to two significant figures. (3 marks)

- 8 The diagram shows part of the graph of  $y = e^{2x} + 3$ .



- (a) Describe a sequence of two geometrical transformations that maps the graph of  $y = e^x$  onto the graph of  $y = e^{2x} + 3$ . (4 marks)
- (b) Use the mid-ordinate rule with four strips of equal width to find an estimate for the area of the shaded region  $A$ , giving your answer to three significant figures. (4 marks)
- (c) Find the exact value of the area of the shaded region  $A$ . (4 marks)
- (d) The region  $B$  is indicated on the diagram. Find the area of the region  $B$ , giving your answer in the form  $pe^8 + qe^4$ , where  $p$  and  $q$  are numbers to be determined. (4 marks)

**END OF QUESTIONS**