General Certificate of Education
January 2007
Advanced Level Examination

MATHEMATICS **MPC3**
Unit Pure Core 3

Thursday 18 January 2007 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 Use the mid-ordinate rule with four strips of equal width to find an estimate for $\int_1^5 \frac{1}{1+\ln x}\,\mathrm{d}x$, giving your answer to three significant figures. *(4 marks)*

2 Describe a sequence of **two** geometrical transformations that maps the graph of $y = \sec x$ onto the graph of $y = 1 + \sec 3x$. *(4 marks)*

3 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = 3 - x^2, \quad \text{for all real values of } x$$

$$\mathrm{g}(x) = \frac{2}{x+1}, \quad \text{for real values of } x,\ x \neq -1$$

(a) Find the range of f. *(2 marks)*

(b) The inverse of g is g^{-1}.

(i) Find $\mathrm{g}^{-1}(x)$. *(3 marks)*

(ii) State the range of g^{-1}. *(1 mark)*

(c) The composite function gf is denoted by h.

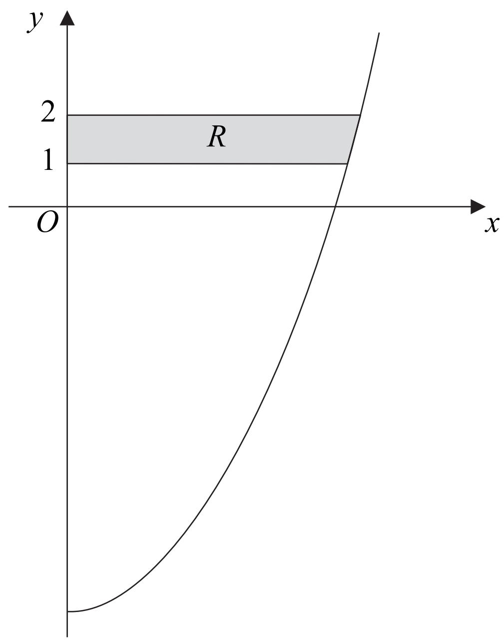
(i) Find $\mathrm{h}(x)$, simplifying your answer. *(2 marks)*

(ii) State the greatest possible domain of h. *(1 mark)*

4 (a) Use integration by parts to find $\int x \sin x \, dx$. *(4 marks)*

(b) Using the substitution $u = x^2 + 5$, or otherwise, find $\int x\sqrt{x^2 + 5} \, dx$. *(4 marks)*

(c) The diagram shows the curve $y = x^2 - 9$ for $x \geqslant 0$.



The shaded region R is bounded by the curve, the lines $y = 1$ and $y = 2$, and the y-axis.

Find the exact value of the volume of the solid generated when the region R is rotated through 360° **about the y-axis**. *(4 marks)*

5 (a) (i) Show that the equation

$$2\cot^2 x + 5\operatorname{cosec} x = 10$$

can be written in the form $2\operatorname{cosec}^2 x + 5\operatorname{cosec} x - 12 = 0$. *(2 marks)*

(ii) Hence show that $\sin x = -\frac{1}{4}$ or $\sin x = \frac{2}{3}$. *(3 marks)*

(b) Hence, or otherwise, solve the equation

$$2\cot^2(\theta - 0.1) + 5\operatorname{cosec}(\theta - 0.1) = 10$$

giving all values of θ in radians to two decimal places in the interval $-\pi < \theta < \pi$. *(3 marks)*

6 (a) Find $\frac{dy}{dx}$ when:

(i) $y = (4x^2 + 3x + 2)^{10}$; *(2 marks)*

(ii) $y = x^2 \tan x$. *(2 marks)*

(b) (i) Find $\frac{dx}{dy}$ when $x = 2y^3 + \ln y$. *(1 mark)*

(ii) Hence find an equation of the tangent to the curve $x = 2y^3 + \ln y$ at the point $(2,1)$. *(3 marks)*

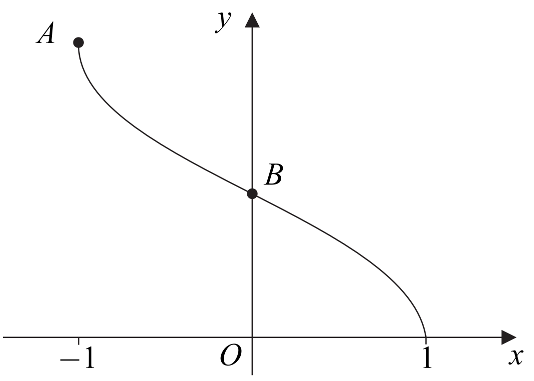
7 (a) Sketch the graph of $y = |2x|$. *(1 mark)*

(b) On a separate diagram, sketch the graph of $y = 4 - |2x|$, indicating the coordinates of the points where the graph crosses the coordinate axes. *(3 marks)*

(c) Solve $4 - |2x| = x$. *(3 marks)*

(d) Hence, or otherwise, solve the inequality $4 - |2x| > x$. *(2 marks)*

8 The diagram shows the curve $y = \cos^{-1} x$ for $-1 \leqslant x \leqslant 1$.

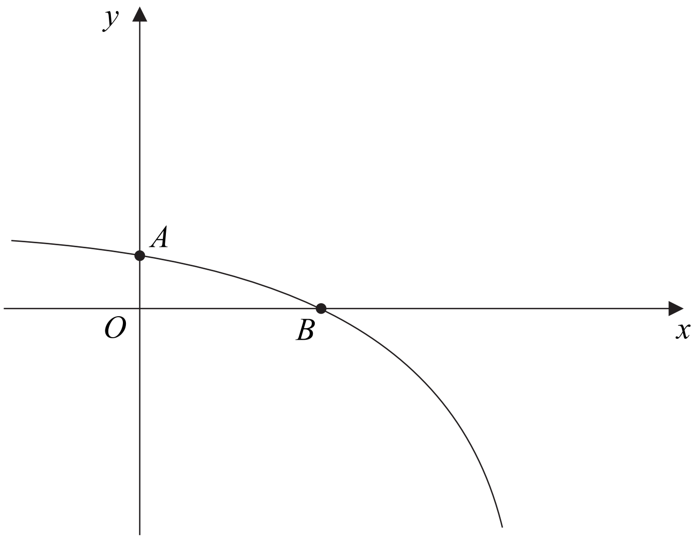


(a) Write down the exact coordinates of the points A and B. *(2 marks)*

(b) The equation $\cos^{-1} x = 3x + 1$ has only one root. Given that the root of this equation is α, show that $0.1 \leqslant \alpha \leqslant 0.2$. *(2 marks)*

(c) Use the iteration $x_{n+1} = \frac{1}{3}(\cos^{-1} x_n - 1)$ with $x_1 = 0.1$ to find the values of x_2, x_3 and x_4, giving your answers to three decimal places. *(3 marks)*

9 The sketch shows the graph of $y = 4 - e^{2x}$. The curve crosses the y-axis at the point A and the x-axis at the point B.



(a) (i) Find $\int (4 - e^{2x})\,dx$. *(2 marks)*

(ii) Hence show that $\int_0^{\ln 2} (4 - e^{2x})\,dx = 4\ln 2 - \frac{3}{2}$. *(2 marks)*

(b) (i) Write down the y-coordinate of A. *(1 mark)*

(ii) Show that $x = \ln 2$ at B. *(2 marks)*

(c) Find the equation of the normal to the curve $y = 4 - e^{2x}$ at the point B. *(4 marks)*

(d) Find the area of the region enclosed by the curve $y = 4 - e^{2x}$, the normal to the curve at B and the y-axis. *(3 marks)*

END OF QUESTIONS

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