

General Certificate of Education  
January 2006  
Advanced Level Examination



**MATHEMATICS**  
**Unit Pure Core 3**

**MPC3**

Wednesday 25 January 2006 9.00 am to 10.30 am

**For this paper you must have:**

- an 8-page answer book
  - the **blue** AQA booklet of formulae and statistical tables
  - an insert for use in Question 6 (enclosed)
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

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Answer **all** questions.

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1 (a) Find  $\frac{dy}{dx}$  when  $y = \tan 3x$ . (2 marks)

(b) Given that  $y = \frac{3x+1}{2x+1}$ , show that  $\frac{dy}{dx} = \frac{1}{(2x+1)^2}$ . (3 marks)

2 Use Simpson's rule with 5 ordinates (4 strips) to find an approximation to

$$\int_1^3 \frac{1}{\sqrt{1+x^3}} dx$$

giving your answer to three significant figures. (4 marks)

3 (a) (i) Given that  $f(x) = x^4 + 2x$ , find  $f'(x)$ . (1 mark)

(ii) Hence, or otherwise, find  $\int \frac{2x^3 + 1}{x^4 + 2x} dx$ . (2 marks)

(b) (i) Use the substitution  $u = 2x + 1$  to show that

$$\int x\sqrt{2x+1} dx = \frac{1}{4} \int \left( u^{\frac{3}{2}} - u^{\frac{1}{2}} \right) du$$
 (3 marks)

(ii) Hence show that  $\int_0^4 x\sqrt{2x+1} dx = 19.9$  correct to three significant figures. (4 marks)

4 It is given that  $2\operatorname{cosec}^2 x = 5 - 5 \cot x$ .

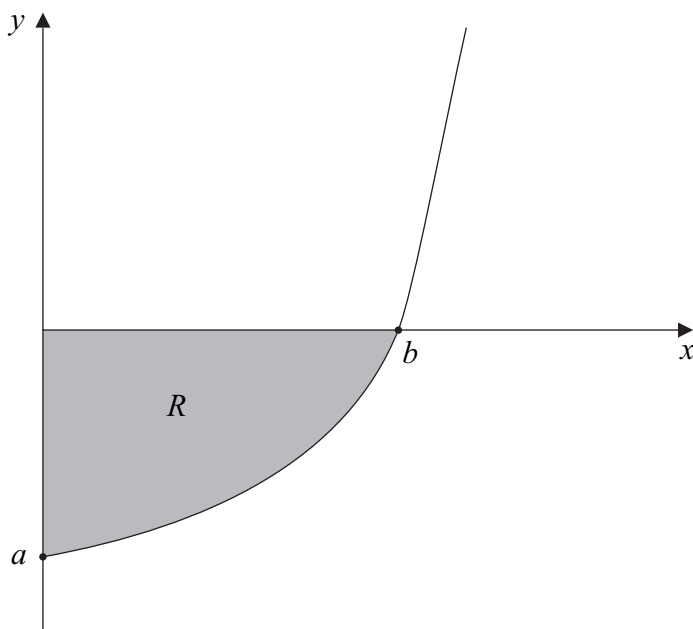
(a) Show that the equation  $2\operatorname{cosec}^2 x = 5 - 5 \cot x$  can be written in the form

$$2 \cot^2 x + 5 \cot x - 3 = 0$$
 (2 marks)

(b) Hence show that  $\tan x = 2$  or  $\tan x = -\frac{1}{3}$ . (2 marks)

(c) Hence, or otherwise, solve the equation  $2\operatorname{cosec}^2 x = 5 - 5 \cot x$ , giving all values of  $x$  in radians to one decimal place in the interval  $-\pi < x \leq \pi$ . (3 marks)

- 5 The diagram shows part of the graph of  $y = e^{2x} - 9$ . The graph cuts the coordinate axes at  $(0, a)$  and  $(b, 0)$ .



- (a) State the value of  $a$ , and show that  $b = \ln 3$ . *(3 marks)*
- (b) Show that  $y^2 = e^{4x} - 18e^{2x} + 81$ . *(1 mark)*
- (c) The shaded region  $R$  is rotated through  $360^\circ$  about the  $x$ -axis. Find the volume of the solid formed, giving your answer in the form  $\pi(p \ln 3 + q)$ , where  $p$  and  $q$  are integers. *(6 marks)*
- (d) Sketch the curve with equation  $y = |e^{2x} - 9|$  for  $x \geq 0$ . *(2 marks)*

**Turn over for the next question**

6 [Figure 1, printed on the insert, is provided for use in this question.]

The curve  $y = x^3 + 4x - 3$  intersects the  $x$ -axis at the point  $A$  where  $x = \alpha$ .

(a) Show that  $\alpha$  lies between 0.5 and 1.0. (2 marks)

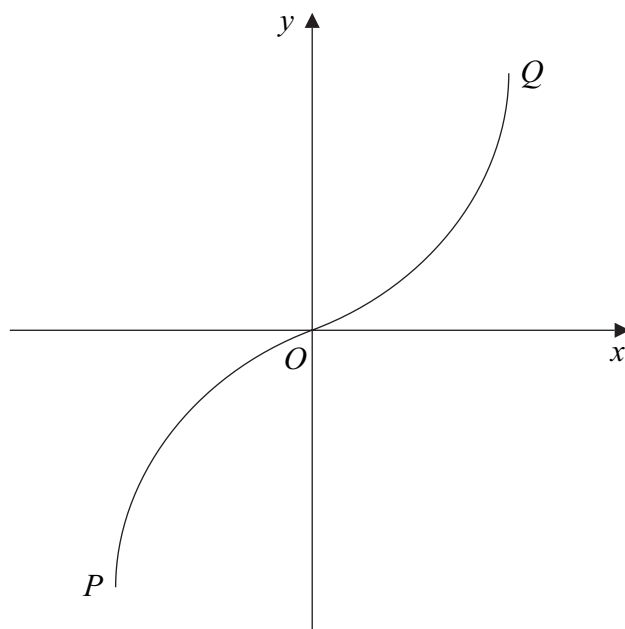
(b) Show that the equation  $x^3 + 4x - 3 = 0$  can be rearranged into the form  $x = \frac{3 - x^3}{4}$ .  
(1 mark)

(c) (i) Use the iteration  $x_{n+1} = \frac{3 - x_n^3}{4}$  with  $x_1 = 0.5$  to find  $x_3$ , giving your answer to two decimal places. (3 marks)

(ii) The sketch on **Figure 1** shows parts of the graphs of  $y = \frac{3 - x^3}{4}$  and  $y = x$ , and the position of  $x_1$ .

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of  $x_2$  and  $x_3$  on the  $x$ -axis. (3 marks)

- 7 (a) The sketch shows the graph of  $y = \sin^{-1} x$ .



Write down the coordinates of the points  $P$  and  $Q$ , the end-points of the graph.

(2 marks)

- (b) Sketch the graph of  $y = -\sin^{-1}(x - 1)$ .

(3 marks)

- 8 The functions  $f$  and  $g$  are defined with their respective domains by

$$f(x) = x^2 \quad \text{for all real values of } x$$

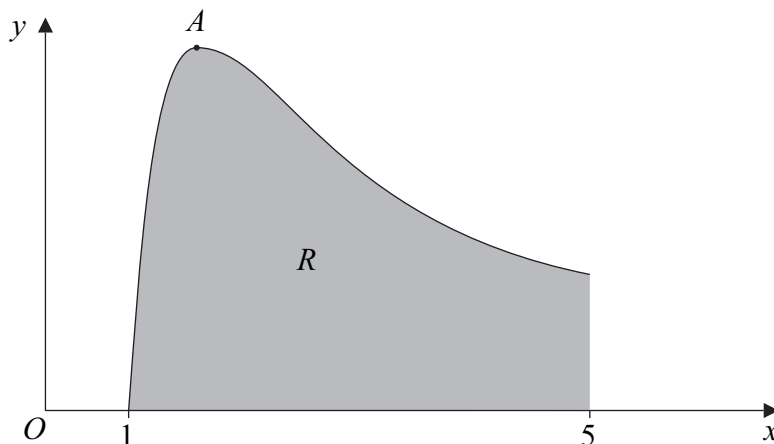
$$g(x) = \frac{1}{x+2} \quad \text{for real values of } x, \quad x \neq -2$$

- (a) State the range of  $f$ . (1 mark)
- (b) (i) Find  $fg(x)$ . (1 mark)
- (ii) Solve the equation  $fg(x) = 4$ . (4 marks)
- (c) (i) Explain why the function  $f$  does **not** have an inverse. (1 mark)
- (ii) The inverse of  $g$  is  $g^{-1}$ . Find  $g^{-1}(x)$ . (3 marks)

9 (a) Given that  $y = x^{-2} \ln x$ , show that  $\frac{dy}{dx} = \frac{1 - 2 \ln x}{x^3}$ . (4 marks)

(b) Using integration by parts, find  $\int x^{-2} \ln x \, dx$ . (4 marks)

(c) The sketch shows the graph of  $y = x^{-2} \ln x$ .



- (i) Using the answer to part (a), find, in terms of  $e$ , the  $x$ -coordinate of the stationary point  $A$ . (2 marks)
- (ii) The region  $R$  is bounded by the curve, the  $x$ -axis and the line  $x = 5$ . Using your answer to part (b), show that the area of  $R$  is

$$\frac{1}{5}(4 - \ln 5) \quad (3 \text{ marks})$$

**END OF QUESTIONS**

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Surname					Other Names				
Centre Number					Candidate Number				
Candidate Signature									

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## Insert

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Insert for use in **Question 6**.

Fill in the boxes at the top of this page.

Attach this insert securely to your answer book.

**Turn over for Figure 1**

**Figure 1 (for Question 6)**