

General Certificate of Education
June 2005
Advanced Subsidiary Examination



MATHEMATICS
Unit Pure Core 2

MPC2

Tuesday 7 June 2005 Afternoon Session

In addition to this paper you will require:

- an 8-page answer book;
 - the **blue** AQA booklet of formulae and statistical tables.
- You **may** use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

Information

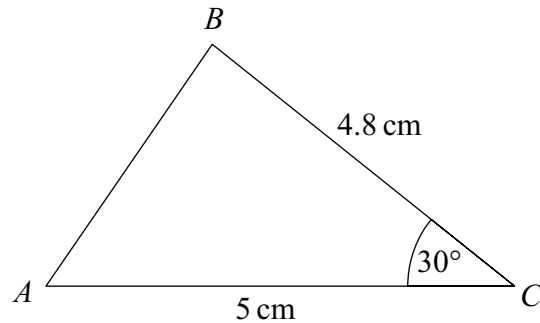
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

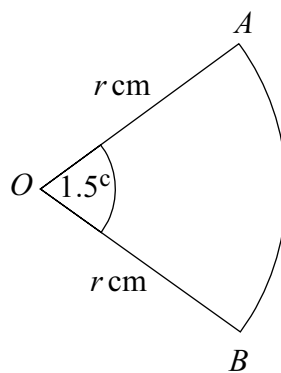
- 1 The diagram shows a triangle ABC .



The lengths of AC and BC are 5 cm and 4.8 cm respectively.

The size of the angle BCA is 30° .

- (a) Calculate the area of the triangle ABC . (2 marks)
- (b) Calculate the length of AB , giving your answer to three significant figures. (3 marks)
- 2 The diagram shows a sector OAB of a circle with centre O and radius $r\text{ cm}$.



The angle AOB is 1.5 radians. The perimeter of the sector is 56 cm .

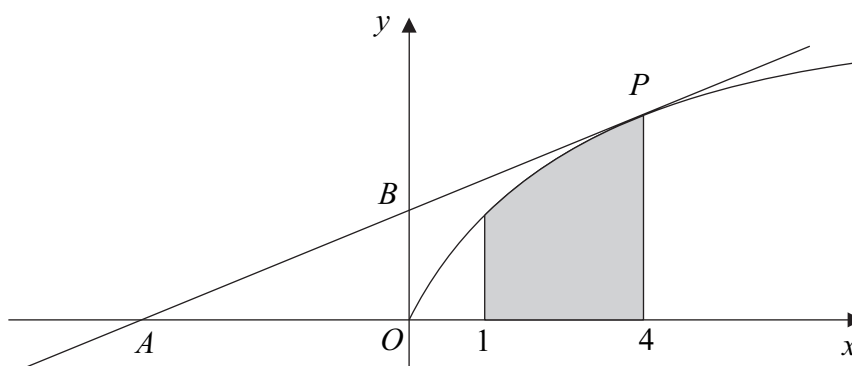
- (a) Show that $r = 16$. (3 marks)
- (b) Find the area of the sector. (2 marks)

3 The n th term of an arithmetic sequence is u_n , where

$$u_n = 90 - 3n$$

- (a) Find the value of u_1 and the value of u_2 . (2 marks)
- (b) Write down the common difference of the arithmetic sequence. (1 mark)
- (c) Given that $\sum_{n=1}^k u_n = 0$, find the value of k . (3 marks)

4 The diagram shows a curve C with equation $y = \sqrt{x}$. The point O is the origin $(0, 0)$.



The region bounded by the curve C , the x -axis and the vertical lines $x = 1$ and $x = 4$ is shown shaded in the diagram.

- (a) (i) Write \sqrt{x} in the form x^p , where p is a constant. (1 mark)
- (ii) Find $\int \sqrt{x} \, dx$. (2 marks)
- (iii) Hence find the area of the shaded region. (3 marks)
- (b) The point on C for which $x = 4$ is P . The tangent to C at the point P intersects the x -axis and the y -axis at the points A and B respectively.
- (i) Find an equation for the tangent to the curve C at the point P . (4 marks)
- (ii) Find the area of the triangle AOB . (3 marks)
- (c) Describe the single geometrical transformation by which the curve with equation $y = \sqrt{x-1}$ can be obtained from the curve C . (2 marks)
- (d) Use the trapezium rule with four ordinates (three strips) to find an approximation for $\int_1^4 \sqrt{x-1} \, dx$, giving your answer to three significant figures. (4 marks)

- 5 The sum to infinity of a geometric series is four times the first term of the series.
- (a) Show that the common ratio, r , of the geometric series is $\frac{3}{4}$. (3 marks)
- (b) The first term of the geometric series is 48. Find the sum of the first 10 terms of the series, giving your answer to four decimal places. (2 marks)
- (c) The n th term of the geometric series is u_n and the $(2n)$ th term of the series is u_{2n} .
- (i) Write u_n and u_{2n} in terms of n . (2 marks)
- (ii) Hence show that $\log_{10}(u_n) - \log_{10}(u_{2n}) = n \log_{10}\left(\frac{4}{3}\right)$. (3 marks)
- (iii) Hence show that the value of

$$\log_{10}\left(\frac{u_{100}}{u_{200}}\right)$$

is 12.5 correct to three significant figures. (2 marks)

- 6 (a) Using the binomial expansion, or otherwise, express $(1+x)^4$ in ascending powers of x . (3 marks)
- (b) (i) Hence show that $(1+\sqrt{5})^4 = 56 + 24\sqrt{5}$. (3 marks)
- (ii) Hence show that $\log_2(1+\sqrt{5})^4 = k + \log_2(7+3\sqrt{5})$, where k is an integer. (3 marks)

- 7 A curve is defined, for $x > 0$, by the equation $y = f(x)$, where

$$f(x) = \frac{x^8 - 1}{x^3}$$

- (a) Express $\frac{x^8 - 1}{x^3}$ in the form $x^p - x^q$, where p and q are integers. (2 marks)
- (b) (i) Hence differentiate $f(x)$ to find $f'(x)$. (2 marks)
- (ii) Hence show that f is an increasing function. (2 marks)
- (c) Find the gradient of the normal to the curve at the point $(1, 0)$. (3 marks)

- 8 (a) (i) Show that the equation

$$4 \tan \theta \sin \theta = 15$$

can be written as

$$4 \sin^2 \theta = 15 \cos \theta \quad (1 \text{ mark})$$

- (ii) Use an appropriate identity to show that the equation

$$4 \sin^2 \theta = 15 \cos \theta$$

can be written as

$$4 \cos^2 \theta + 15 \cos \theta - 4 = 0 \quad (2 \text{ marks})$$

- (b) (i) Solve the equation $4c^2 + 15c - 4 = 0$. (2 marks)

- (ii) Hence explain why the only value of $\cos \theta$ which satisfies the equation

$$4 \cos^2 \theta + 15 \cos \theta - 4 = 0$$

is $\cos \theta = \frac{1}{4}$. (1 mark)

- (iii) Hence solve the equation $4 \tan \theta \sin \theta = 15$ giving all solutions to the nearest 0.1° in the interval $0^\circ \leq \theta \leq 360^\circ$. (2 marks)

- (c) **Write down** all the values of x in the interval $0^\circ \leq x \leq 90^\circ$ for which

$$4 \tan 4x \sin 4x = 15$$

giving your answers to the nearest degree. (2 marks)

END OF QUESTIONS

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE