

General Certificate of Education  
January 2005  
Advanced Subsidiary Examination



**MATHEMATICS**  
**Unit Pure Core 2**

**MPC2**

Friday 21 January 2005 Afternoon Session

**In addition to this paper you will require:**

- an 8-page answer book;
  - the **blue** AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

**Information**

- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

**Advice**

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

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Answer **all** questions.

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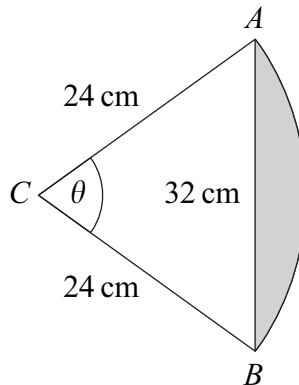
1 A curve is defined for  $x > 0$  by the equation  $y = x + \frac{2}{x}$ .

(a) (i) Find  $\frac{dy}{dx}$ . (3 marks)

(ii) Hence show that the gradient of the curve at the point  $P$  where  $x = 2$  is  $\frac{1}{2}$ . (1 mark)

(b) Find an equation of the normal to the curve at this point  $P$ . (4 marks)

2 The diagram shows a triangle  $ABC$  and the arc  $AB$  of a circle whose centre is  $C$  and whose radius is 24 cm.



The length of the side  $AB$  of the triangle is 32 cm. The size of the angle  $ACB$  is  $\theta$  radians.

(a) Show that  $\theta = 1.46$  correct to three significant figures. (3 marks)

(b) Calculate the length of the arc  $AB$  to the nearest cm. (2 marks)

(c) (i) Calculate the area of the sector  $ABC$  to the nearest  $\text{cm}^2$ . (2 marks)

(ii) Hence calculate the area of the shaded segment to the nearest  $\text{cm}^2$ . (3 marks)

3 An arithmetic series has fifth term 46 and twentieth term 181.

- (a) (i) Show that the common difference is 9. (3 marks)
- (ii) Find the first term. (1 mark)
- (b) Find the sum of the first 20 terms of the series. (2 marks)
- (c) The  $n$ th term of the series is  $u_n$ .

Given that the sum of the first 50 terms of the series is 11525, find the value of

$$\sum_{n=21}^{50} u_n \quad (2 \text{ marks})$$

- 4 (a) Write  $\sqrt{x}$  in the form  $x^k$ , where  $k$  is a fraction. (1 mark)
- (b) Hence express  $\sqrt{x}(x-1)$  in the form  $x^p - x^q$ . (2 marks)
- (c) Find  $\int \sqrt{x}(x-1) dx$ . (3 marks)
- (d) Hence show that  $\int_1^2 \sqrt{x}(x-1) dx = \frac{4}{15}(\sqrt{2} + 1)$ . (3 marks)

5 (a) Given that

$$\log_a x = 3 \log_a 6 - \log_a 8$$

where  $a$  is a positive constant, show that  $x = 27$ . (3 marks)

(b) Write down the value of:

- (i)  $\log_4 1$ ; (1 mark)
- (ii)  $\log_4 4$ ; (1 mark)
- (iii)  $\log_4 2$ ; (1 mark)
- (iv)  $\log_4 8$ . (1 mark)

6 (a) (i) Using the binomial expansion, or otherwise, express  $(2+x)^3$  in the form  $8+ax+bx^2+x^3$ , where  $a$  and  $b$  are integers. (3 marks)

(ii) Write down the expansion of  $(2-x)^3$ . (2 marks)

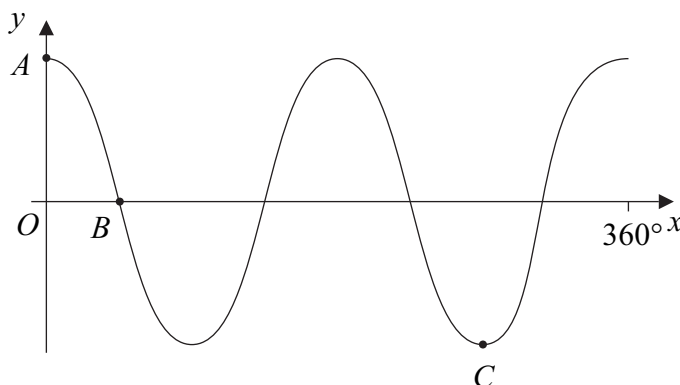
(b) Hence show that  $(2+x)^3 - (2-x)^3 = 24x + 2x^3$ . (2 marks)

(c) Hence show that the curve with equation

$$y = (2+x)^3 - (2-x)^3$$

has no stationary points. (3 marks)

7 The diagram shows the graph of  $y = \cos 2x$  for  $0^\circ \leq x \leq 360^\circ$ .



(a) Write down the coordinates of the points  $A$ ,  $B$  and  $C$  marked on the diagram. (4 marks)

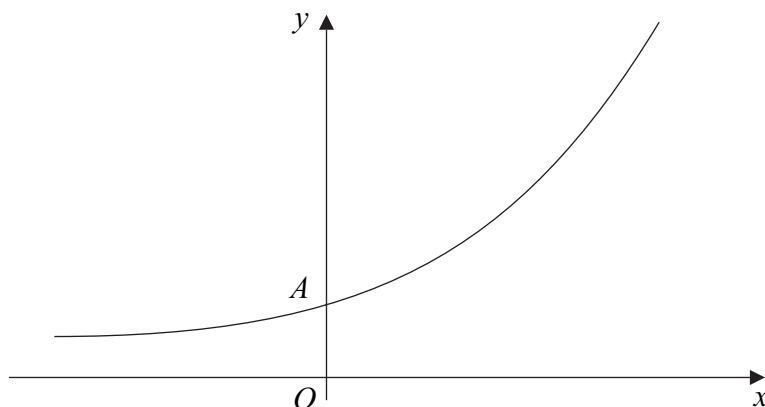
(b) Describe the single geometrical transformation by which the curve with equation  $y = \cos 2x$  can be obtained from the curve with equation  $y = \cos x$ . (2 marks)

(c) Solve the equation

$$\cos 2x = 0.37$$

giving all solutions to the nearest  $0.1^\circ$  in the interval  $0^\circ \leq x \leq 360^\circ$ . (No credit will be given for simply reading values from a graph.) (5 marks)

- 8 The diagram shows a sketch of the curve with equation  $y = 3^x + 1$ .



The curve intersects the  $y$ -axis at the point  $A$ .

- (a) Write down the  $y$ -coordinate of point  $A$ . *(1 mark)*
- (b) (i) Use the trapezium rule with five ordinates (four strips) to find an approximation for  $\int_0^1 (3^x + 1) dx$ , giving your answer to three significant figures. *(4 marks)*
- (ii) By considering the graph of  $y = 3^x + 1$ , explain with the aid of a diagram whether your approximation will be an overestimate or an underestimate of the true value of  $\int_0^1 (3^x + 1) dx$ . *(2 marks)*
- (c) The line  $y = 5$  intersects the curve  $y = 3^x + 1$  at the point  $P$ . By solving a suitable equation, find the  $x$ -coordinate of the point  $P$ . Give your answer to four decimal places. *(4 marks)*
- (d) The curve  $y = 3^x + 1$  is reflected in the  $y$ -axis to give the curve with equation  $y = f(x)$ . Write down an expression for  $f(x)$ . *(1 mark)*

**END OF QUESTIONS**

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