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ASSESSMENT and QUALIFICATIONS ALLIANCE

# General Certificate of Education

# Mathematics 6360

*MPC2 Pure Core 2*

# Mark Scheme

## *2005 examination – June series*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

# Key to mark scheme and abbreviations used in marking

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |

| | | | |
|---|---|---|---|
| √ or ft or F | follow through from previous incorrect result | MC | mis-copy |
| CAO | correct answer only | MR | mis-read |
| CSO | correct solution only | RA | required accuracy |
| AWFW | anything which falls within | FW | further work |
| AWRT | anything which rounds to | ISW | ignore subsequent work |
| ACF | any correct form | FIW | from incorrect work |
| AG | answer given | BOD | given benefit of doubt |
| SC | special case | WR | work replaced by candidate |
| OE | OE | FB | formulae book |
| A2,1 | 2 or 1 (or 0) accuracy marks | NOS | not on scheme |
| $-x$ EE | deduct $x$ marks for each error | G | graph |
| NMS | no method shown | c | candidate |
| PI | possibly implied | sf | significant figure(s) |
| SCA | substantially correct approach | dp | decimal place(s) |

## Application of Mark Scheme

**No method shown:**

| | |
|---|---|
| Correct answer without working | mark as in scheme |
| Incorrect answer without working | zero marks unless specified otherwise |

**More than one method / choice of solution:**

| | |
|---|---|
| 2 or more complete attempts, neither/none crossed out | mark both/all fully and award the mean mark rounded down |
| 1 complete and 1 partial attempt, neither crossed out | award credit for the complete solution only |

| | |
|---|---|
| **Crossed out work** | do not mark unless it has not been replaced |
| **Alternative solution** using a correct or partially correct method | award method and accuracy marks as appropriate |

**MPC2**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 1(a) | Area = $\frac{1}{2}\times 5\times 4.8\times \sin 30^{\circ}$ | M1 | | Use of $\frac{1}{2}ab\sin C$ OE |
| | = 6 cm$^2$. | A1 | 2 | Condone absent cm$^2$. [**Note:** Calculator set in wrong mode, penalise only once on the paper.] |
| (b) | $AB^2 = 5^2 + 4.8^2 - 2\times 5\times 4.8\cos 30^{\circ}$ | M1 | | RHS of cosine rule used |
| | $= 25 + 23.04 - 41.569$ | m1 | | Correct order of evaluation |
| | = 6.4707.. <br> $\Rightarrow$ AB = $\sqrt{6.47...} = 2.5437$ <br> = 2.54 cm to 3 sf | A1 | 3 | Accept 'better' than 2.54 <br> Condone absent cm |
| | **Total** | | **5** | |
| 2(a) | Arc = $r\theta$ | M1 | | For $r\theta$ or $16\theta$ or $16\times 1.5$ OE multiplicatio |
| | $1.5r + r + r$ (= 56) | M1 | | For realising that perimeter is sum of two radii and arc. |
| | $3.5r = 56 \Rightarrow r = 16$ | A1 | 3 | **AG** Completion (condone verification) |
| (b) | Area of sector = $\frac{1}{2}r^2\theta$ | M1 | | $\frac{1}{2}r^2\,\theta$ OE seen |
| | $= \frac{1}{2}16^2(1.5) = 192$ cm$^2$. | A1 | 2 | Condone absent cm$^2$. |
| | **Total** | | **5** | |
| 3(a) | $u_1 = 87$; $u_2 = 84$ | B1;B1 √ | 2 | ft on $u_2 = u_1 - 3$ **SC** B1 for 90, 87 |
| (b) | Common difference ($d$) is – 3 | B1 | 1 | |
| (c) | $\sum_{n=1}^{k} u_n$ = sum of AP | M1 | | |
| | ……..= $\frac{k}{2}[174 + (k-1)(-3)]$ | A1√ | | OE ft on $u_1$ and use of $d$ = 3 <br> (For M1A1 ft condone $n$ in place of $k$) |
| | $0 = \frac{k}{2}[177 - 3k] \Rightarrow 177 = 3k$ <br> $\Rightarrow k = 59$ | A1 | 3 | Just the **single** value 59 |
| ALTI | $= \sum_{n=1}^{k} 90 - \sum_{n=1}^{k} 3n = 90k - 3\left[\frac{k}{2}(k+1)\right]$ | M1;A1 | | M1 split and either $90k$ or $\left[\frac{k}{2}(k+1)\right]$ |
| | $0 = 90k - 1.5k(k+1) \Rightarrow k = 59$ | A1 | | (For 1st two marks condone $n$ in place of $k$) |
| | **Total** | | **6** | |

**MPC2 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **4(a)(i)** | $\sqrt{x} = x^{\frac{1}{2}}$ | B1 | 1 | Accept $p = 0.5$ |
| **(ii)** | $\int \sqrt{x}\ \mathrm{d}x = \frac{x^{1.5}}{1.5}\{+c\}$ | M1<br>A1√ | 2 | Index raised by 1<br>Correct ft on $p$. Condone missing '+c' |
| **(iii)** | Area = $\int_1^4 \sqrt{x}\ \mathrm{d}x$ | B1 | | Limits 1 and 4 PI |
| | $\ldots\ldots = \frac{4^{1.5}}{1.5} - \frac{1}{1.5}$ | M1 | | F(4) – F(1) |
| | $= \frac{14}{3}$ | A1 | 3 | Accept 4.66 or better |
| **(b)(i)** | $y = x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ | M1 | | Index ( $p$–1 ) ft |
| | When $x$= 4, $y'(4) = 0.25$ | M1 | | Attempt to find $y'(4)$. |
| | When $x$= 4, $y = 2$ | B1 | | |
| | Equation of tangent: $y - 2 = \frac{1}{4}(x-4)$ | A1 | 4 | accept other forms |
| **(ii)** | When $x = 0$, $y = 1$ $B(0, 1)$ | M1 | | Subs $x = 0$ **and** then $y = 0$ into equation |
| | When $y = 0$, $x = -4$ $A(-4, 0)$ | A1√ | | of tangent. PI<br>Correct ft $y_B$ and $x_A$<br>(may be awarded as part of area calculation) |
| | Area = 0.5(1)(4) = 2 | A1√ | 3 | ft wrong sloping tangent and max of 1 further slip. Final answer must be +'ve |
| **(c)** | Translation | B1 | | 'Translation'/'translate(d)' |
| | $\begin{bmatrix}1\\0\end{bmatrix}$ | B1 | 2 | Accept equivalent in words provided linked to 'translation/move/shift'<br>(Note: B0B1 is possible) |
| **(d)** | $h = 1$<br>Integral = $h/2$ {……} | B1 | | PI |
| | $\{\ldots.\} = \mathrm{f}(1) + 2[\mathrm{f}(2) + \mathrm{f}(3)] + \mathrm{f}(4)$ | M1 | | OE summing of areas of the three traps |
| | $\{\ldots.\} = 0 + 2\left(1 + \sqrt{2}\right) + \sqrt{3}$ | A1 | | Condone 1 numerical slip |
| | Integral = $\frac{1}{2}\{\ 2(1+1.414...)+1.732\ \}$ | | | |
| | Integral = $0.5 \times 6.560\ldots$ =3.28 to 3sf | A1 | 4 | CAO Must be 3.28 |
| | **Total** | | **19** | |

**MPC2 (Cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **5(a)** | $\frac{a}{1-r}=4a$ | M1 | | (Accept $S_\infty=\frac{a}{1-\frac{3}{4}}$) |
| | $\Rightarrow 1-r=\frac{a}{4a}$ or $a=4a(1-r)$ | A1 | | Either (or better) (or $S_\infty=4a$ if M1 above) |
| | $1-r=\frac{1}{4}\Rightarrow r=\frac{3}{4}$ | A1 | 3 | **AG** CSO Be convinced. (or statement 4 times 1st term) |
| **(b)** | $(S_{10}=)\frac{48(1-r^{10})}{1-r}$ | M1 | | Correct formula with n = 10 and one of $a=48$ or $r=\frac{3}{4}$ OE |
| | $=192(1–0.75^{10})=181.1878$ to 4dp | A1 | 2 | |
| **(c)(i)** | $u_n=\underline{ar^{n-1}}=a\left(\frac{3}{4}\right)^{n-1}=48\left(\frac{3}{4}\right)^{n-1}$ | B1 | | |
| | $u_{2n}=\underline{ar^{2n-1}}=a\left(\frac{3}{4}\right)^{2n-1}=48\left(\frac{3}{4}\right)^{2n-1}$ | B1√ | 2 | ft on candidate's $u_n=ar^{\text{function of } n}$ |
| **(ii)** | $\frac{u_n}{u_{2n}}=\frac{ar^{n-1}}{ar^{2n-1}}=\frac{r^{n-1}}{r^{2n-1}}$ | M1 | | Eliminating $a$ (or 48) or log $a$ |
| | $\log_{10}u_n-\log_{10}u_{2n}=\log_{10}\frac{u_n}{u_{2n}}$ | M1 | | Using at least one log law |
| | $=\log_{10}\frac{r^{n-1}}{r^{2n-1}}=\log_{10}\left(r^{-n}\right)$ | | | |
| | $=-n\log_{10}\frac{3}{4}=n\log_{10}\frac{4}{3}$ | A1 | 3 | **AG** CSO Full valid completion |
| **(iii)** | $\log_{10}\left[\frac{u_{100}}{u_{200}}\right]=100\log_{10}\left(\frac{4}{3}\right)$ | M1 | | |
| | = 12.49…. = 12.5 to 3 sf | A1 | 2 | **AG** CSO Be convinced<br>**SC:**Those applying 'hence' to (i) rather than to (ii)<br>**Mark as B2** |
| | **Total** | | **12** | |

**MPC2 (Cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 6(a) | $(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$ | M1<br>A2,1 | 3 | Full method<br>A1 if four terms correct or just one slip |
| (b)(i) | $(1+\sqrt{5})^4 = 1 + 4\sqrt{5} + 6(\sqrt{5})^2 + 4(\sqrt{5})^3 + (\sqrt{5})^4$ | M1 | | Substitute. √5 for $x$. |
| | $= 1 + 4\sqrt{5} + 6(5) + 4(5\sqrt{5}) + (25)$ | A1ft | | Two of 3 terms shown in brackets |
| | ……….= $56 + 24\sqrt{5}$ | A1 | 3 | **AG** CSO Be convinced |
| (ii) | $\log_2 (1+\sqrt{5})^4 = \log_2[8(7+3\sqrt{5})]$ | M1 | | |
| | $= \log_2 8 + \log_2(7+3\sqrt{5})$ | m1 | | |
| | $= 3 + \log_2(7+3\sqrt{5})$ | A1 | 3 | CSO<br>**SC** B1 Change to base 10 and verify |
| | **Total** | | **9** | |
| 7(a) | ……..= $x^5 - x^{-3}$ | M1<br>A1 | 2 | One power correct<br>Accept $p = 5$, $q = -3$ |
| (b)(i) | $f'(x) = 5x^4$<br>$+3x^{-4}$ | B1√<br>B1√ | 2 | ft on $px^{p-1}$<br>ft on $-qx^{q-1}$ provided $q < 0$ |
| (ii) | $f'(x) \left\{= 5x^4 + \frac{3}{x^4}\right\} > 0$ | M1 | | M1 Considers sign of $f'(x)$; a statement "$f'(x) > 0$ OE" with 'f increasing'. |
| | ⇒ f is increasing {function} | A1 | 2 | A1 needs $f'(x)$ of the form $ax^4 + \frac{b}{x^4}$, where a and b both > 0 and no incorrect statements based on $f'(x)$ at different values of $x$ |
| (c) | At (1,0), $f'(1) = 5 + 3 = 8$ | M1 | | Attempts to find $f'(1)$ |
| | ⇒ grad. of normal = $-\frac{1}{8}$ | m1<br>A1√ | 3 | Use of $m \times m' = -1$ PI<br>ft on wrong $f'(x)$ |
| | **Total** | | **9** | |

**MPC2 (Cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **8(a)(i)** | $4\dfrac{\sin\theta}{\cos\theta}\sin\theta = 15$ <br> $\Rightarrow 4\sin^2\theta = 15\cos\theta$ | B1 | 1 | **AG** Be convinced |
| **(ii)** | $\sin^2\theta + \cos^2\theta = 1$ | M1 | | OE seen |
| | $4(1-\cos^2\theta) = 15\cos\theta$ <br> $4\cos^2\theta + 15\cos\theta - 4 = 0$ | A1 | 2 | **AG** Be convinced |
| **(b)(i)** | $(4c-1)(c+4) = 0$ | M1 | | Factorisation or formula or completion of square |
| | $c = -4,\ \ c = \dfrac{1}{4}$ | A1 | 2 | Both values |
| **(ii)** | Since $-1 \le \cos\theta\ \{\le 1\}$, the only possible value for $\cos\theta$ is $\dfrac{1}{4}$ | E1√ | 1 | **AG** convincingly explained (Condone strict inequalities) Ft provided candidates answers for $c$ are $\dfrac{1}{4}$ and a value $k$ such that $k > 1$ or $k < -1$ |
| **(iii)** | $\theta = 75.5°$ | B1 | | |
| | $\theta = 284.5°$ | B1√ | 2 | Ft on [360 – c's 75.5° ] as only other solution in the given interval |
| **(c)** | $\ldots\ldots\ldots \Rightarrow \cos 4x = \dfrac{1}{4}$ | M1 | | Links with previous parts. PI |
| | $x = 19°, 71°$ | A1√ | 2 | Ft on (iii)/4…..(only ft if 2 answers in given range). |
| | **Total** | | **10** | |
| | **Total** | | **75** | |