

General Certificate of Education
June 2008
Advanced Subsidiary Examination



MATHEMATICS
Unit Pure Core 1

MPC1

Thursday 15 May 2008 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
 - the blue AQA booklet of formulae and statistical tables.
- You must **not** use a calculator.



Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 The straight line L has equation $y = 3x - 1$ and the curve C has equation

$$y = (x + 3)(x - 1)$$

- (a) Sketch on the same axes the line L and the curve C , showing the values of the intercepts on the x -axis and the y -axis. (5 marks)
- (b) Show that the x -coordinates of the points of intersection of L and C satisfy the equation $x^2 - x - 2 = 0$. (2 marks)
- (c) Hence find the coordinates of the points of intersection of L and C . (4 marks)

- 2 It is given that $x = \sqrt{3}$ and $y = \sqrt{12}$.

Find, in the simplest form, the value of:

- (a) xy ; (1 mark)
- (b) $\frac{y}{x}$; (2 marks)
- (c) $(x + y)^2$. (3 marks)

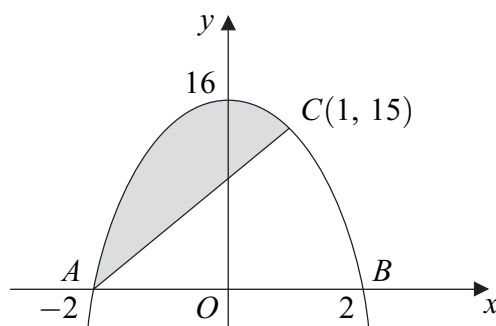
- 3 Two numbers, x and y , are such that $3x + y = 9$, where $x \geq 0$ and $y \geq 0$.

It is given that $V = xy^2$.

- (a) Show that $V = 81x - 54x^2 + 9x^3$. (2 marks)
- (b) (i) Show that $\frac{dV}{dx} = k(x^2 - 4x + 3)$, and state the value of the integer k . (4 marks)
- (ii) Hence find the two values of x for which $\frac{dV}{dx} = 0$. (2 marks)
- (c) Find $\frac{d^2V}{dx^2}$. (2 marks)
- (d) (i) Find the value of $\frac{d^2V}{dx^2}$ for each of the two values of x found in part (b)(ii). (1 mark)
- (ii) Hence determine the value of x for which V has a maximum value. (1 mark)
- (iii) Find the maximum value of V . (1 mark)

- 4 (a) Express $x^2 - 3x + 4$ in the form $(x - p)^2 + q$, where p and q are rational numbers. (2 marks)
- (b) Hence write down the minimum value of the expression $x^2 - 3x + 4$. (1 mark)
- (c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 - 3x + 4$. (3 marks)

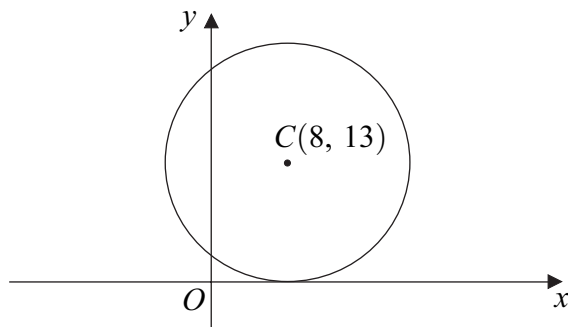
- 5 The curve with equation $y = 16 - x^4$ is sketched below.



The points $A(-2, 0)$, $B(2, 0)$ and $C(1, 15)$ lie on the curve.

- (a) Find an equation of the straight line AC . (3 marks)
- (b) (i) Find $\int_{-2}^1 (16 - x^4) \, dx$. (5 marks)
- (ii) Hence calculate the area of the shaded region bounded by the curve and the line AC . (3 marks)
- 6 The polynomial $p(x)$ is given by $p(x) = x^3 + x^2 - 8x - 12$.
- (a) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x - 1$. (2 marks)
- (b) (i) Use the Factor Theorem to show that $x + 2$ is a factor of $p(x)$. (2 marks)
- (ii) Express $p(x)$ as the product of linear factors. (3 marks)
- (c) (i) The curve with equation $y = x^3 + x^2 - 8x - 12$ passes through the point $(0, k)$. State the value of k . (1 mark)
- (ii) Sketch the graph of $y = x^3 + x^2 - 8x - 12$, indicating the values of x where the curve touches or crosses the x -axis. (3 marks)

- 7 The circle S has centre $C(8, 13)$ and touches the x -axis, as shown in the diagram.



- (a) Write down an equation for S , giving your answer in the form

$$(x - a)^2 + (y - b)^2 = r^2 \quad (2 \text{ marks})$$

- (b) The point P with coordinates $(3, 1)$ lies on the circle.

- (i) Find the gradient of the straight line passing through P and C . (1 mark)
- (ii) Hence find an equation of the tangent to the circle S at the point P , giving your answer in the form $ax + by = c$, where a , b and c are integers. (4 marks)
- (iii) The point Q also lies on the circle S , and the length of PQ is 10. Calculate the shortest distance from C to the chord PQ . (3 marks)

- 8 The quadratic equation $(k + 1)x^2 + 4kx + 9 = 0$ has real roots.

- (a) Show that $4k^2 - 9k - 9 \geq 0$. (3 marks)
- (b) Hence find the possible values of k . (4 marks)

END OF QUESTIONS