General Certificate of Education
June 2007
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

Monday 21 May 2007 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The points A and B have coordinates $(6, -1)$ and $(2, 5)$ respectively.

(a) (i) Show that the gradient of AB is $-\frac{3}{2}$. *(2 marks)*

(ii) Hence find an equation of the line AB, giving your answer in the form $ax + by = c$, where a, b and c are integers. *(2 marks)*

(b) (i) Find an equation of the line which passes through B and which is perpendicular to the line AB. *(2 marks)*

(ii) The point C has coordinates $(k, 7)$ and angle ABC is a right angle.

Find the value of the constant k. *(2 marks)*

2 (a) Express $\frac{\sqrt{63}}{3} + \frac{14}{\sqrt{7}}$ in the form $n\sqrt{7}$, where n is an integer. *(3 marks)*

(b) Express $\frac{\sqrt{7}+1}{\sqrt{7}-2}$ in the form $p\sqrt{7} + q$, where p and q are integers. *(4 marks)*

3 (a) (i) Express $x^2 + 10x + 19$ in the form $(x + p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Write down the coordinates of the vertex (minimum point) of the curve with equation $y = x^2 + 10x + 19$. *(2 marks)*

(iii) Write down the equation of the line of symmetry of the curve $y = x^2 + 10x + 19$. *(1 mark)*

(iv) Describe geometrically the transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 10x + 19$. *(3 marks)*

(b) Determine the coordinates of the points of intersection of the line $y = x + 11$ and the curve $y = x^2 + 10x + 19$. *(4 marks)*

4 A model helicopter takes off from a point O at time $t = 0$ and moves vertically so that its height, y cm, above O after time t seconds is given by

$$y = \frac{1}{4}t^4 - 26t^2 + 96t\,, \qquad 0 \leqslant t \leqslant 4$$

(a) Find:

(i) $\dfrac{\mathrm{d}y}{\mathrm{d}t}$; *(3 marks)*

(ii) $\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}$. *(2 marks)*

(b) Verify that y has a stationary value when $t = 2$ and determine whether this stationary value is a maximum value or a minimum value. *(4 marks)*

(c) Find the rate of change of y with respect to t when $t = 1$. *(2 marks)*

(d) Determine whether the height of the helicopter above O is increasing or decreasing at the instant when $t = 3$. *(2 marks)*

5 A circle with centre C has equation $(x + 3)^2 + (y - 2)^2 = 25$.

(a) Write down:

(i) the coordinates of C ; *(2 marks)*

(ii) the radius of the circle. *(1 mark)*

(b) (i) Verify that the point $N(0, -2)$ lies on the circle. *(1 mark)*

(ii) Sketch the circle. *(2 marks)*

(iii) Find an equation of the normal to the circle at the point N. *(3 marks)*

(c) The point P has coordinates (2, 6).

(i) Find the distance PC, leaving your answer in surd form. *(2 marks)*

(ii) Find the length of a tangent drawn from P to the circle. *(3 marks)*

Turn over for the next question

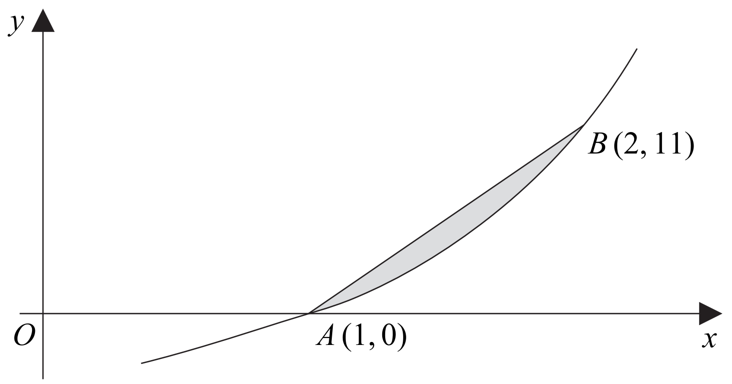
6 (a) The polynomial $\mathrm{f}(x)$ is given by $\mathrm{f}(x) = x^3 + 4x - 5$.

(i) Use the Factor Theorem to show that $x - 1$ is a factor of $\mathrm{f}(x)$. *(2 marks)*

(ii) Express $\mathrm{f}(x)$ in the form $(x-1)(x^2 + px + q)$, where p and q are integers. *(2 marks)*

(iii) Hence show that the equation $\mathrm{f}(x) = 0$ has exactly one real root and state its value. *(3 marks)*

(b) The curve with equation $y = x^3 + 4x - 5$ is sketched below.



The curve cuts the x-axis at the point $A(1, 0)$ and the point $B(2, 11)$ lies on the curve.

(i) Find $\int (x^3 + 4x - 5)\,\mathrm{d}x$. *(3 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line AB. *(4 marks)*

7 The quadratic equation

$$(2k-3)x^2 + 2x + (k-1) = 0$$

where k is a constant, has real roots.

(a) Show that $2k^2 - 5k + 2 \leqslant 0$. *(3 marks)*

(b) (i) Factorise $2k^2 - 5k + 2$. *(1 mark)*

(ii) Hence, or otherwise, solve the quadratic inequality

$$2k^2 - 5k + 2 \leqslant 0$$ *(3 marks)*

END OF QUESTIONS